Responses on Sample Covariance

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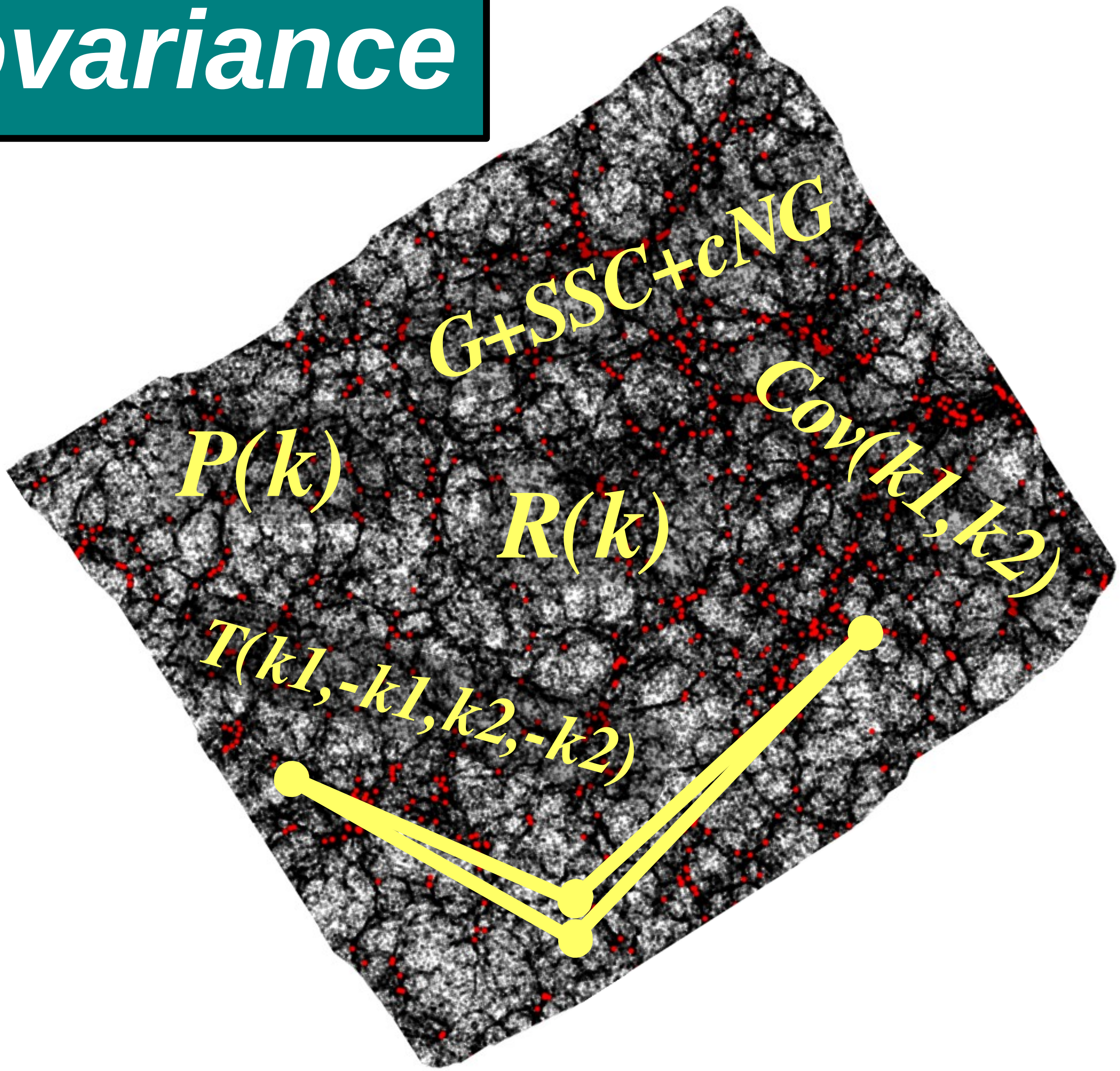
MPA

with Elisabeth Krause & Fabian Schmidt



arXiv:1703.09212
arXiv:1705.01092
arXiv:1711.07467

Statistical challenges for large-scale structure in the era of LSST
Oxford 2018

Covariances in our life

- The Gaussian **likelihood** of a certain set of parameters given a **hypothetical** survey measurement of the 3D matter power spectrum ***P(k)***:

$$\mathcal{L}(\theta) \propto \exp\left[\sum_{i,j}\left(P_m^{\text{theory}}(k_i,\theta) - P_m^{\text{data}}(k_i)\right)\text{Cov}^{-1}(k_i,k_j)\left(P_m^{\text{theory}}(k_j,\theta) - P_m^{\text{data}}(k_j)\right)\right]$$

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Measured data

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Measured data

Theoretical prediction

$$\mathcal{L}(\theta) \propto \exp\left[\sum_{i,j}\left(P_m^{\rm theory}(k_i,\theta) - P_m^{\rm data}(k_i)\right){\rm Cov}^{-1}(k_i,k_j)\left(P_m^{\rm theory}(k_j,\theta) - P_m^{\rm data}(k_j)\right)\right]$$

Covariances in our life

- The Gaussian **likelihood** of a certain set of parameters given a **hypothetical** survey measurement of the 3D matter power spectrum ***P(k)***:

Measured data — *Theoretical prediction*

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Covariance matrix

- **Don't know how to compute it accurately/efficiently;**
- By far, the **least well understood piece of this likelihood**: what is its redshift and cosmological dependence; baryonic effects?

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Covariance matrix

We'll address this!

- **Don't know how to compute it accurately/efficiently:**
- By far, the **least well understood piece of this likelihood**: what is its redshift and cosmological dependence; baryonic effects?

In this talk ...

1) Response Approach to Perturbation Theory

Barreira, Schmidt , 1703.09212

2) An application to the lensing covariance

Barreira, Schmidt , 1705.01092

Barreira, Krause, Schmidt, 1711.07467

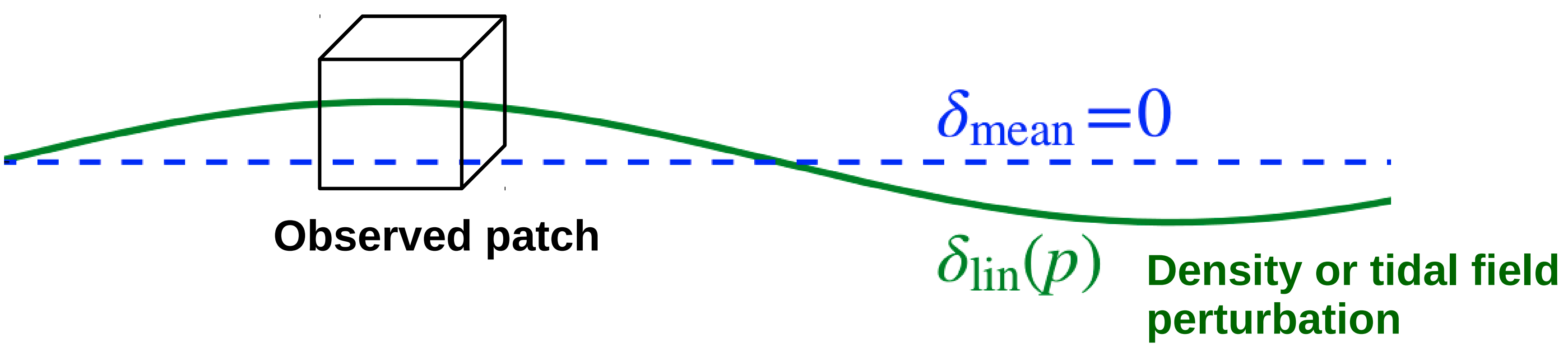
Response Approach to PT

Barreira, Schmidt , 1703.09212

What are responses?

Responses describe how the power spectrum responds to the presence of large-scale perturbations.

$$\mathcal{R}_n \equiv \frac{1}{n! P(k)} \frac{\mathrm{d}^n P(\boldsymbol{k}, \delta_1 \cdots \delta_n)}{\mathrm{d}\delta_1 \cdots \mathrm{d}\delta_n} \bigg|_{\delta_a = 0}$$



What are responses?

Responses describe how the power spectrum responds to the presence

turbations.

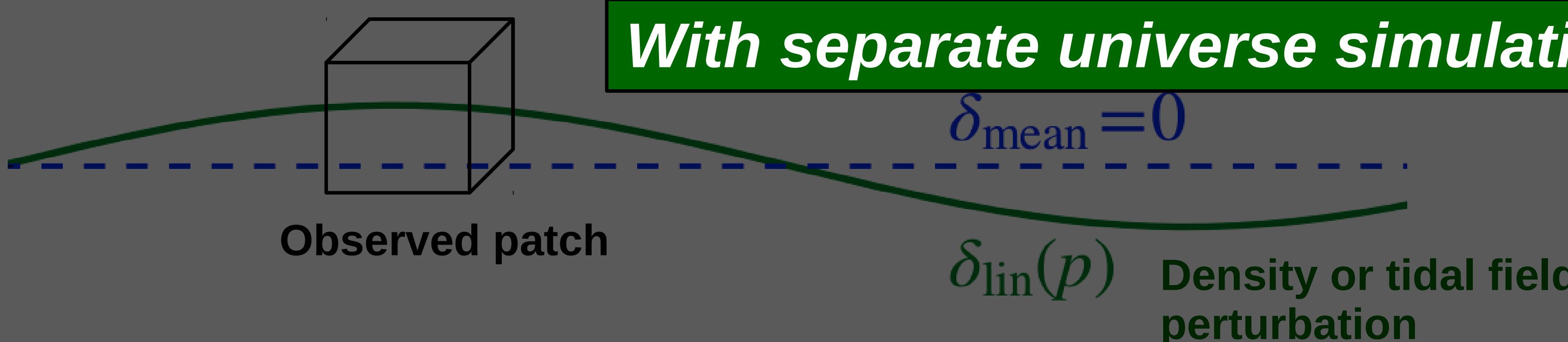
What are they good for?

To describe squeezed N-point functions

$$\mathcal{R}_n \equiv \frac{1}{n!P(k)} \frac{\mathrm{d}^n P(\boldsymbol{k}, \delta_1 \cdots \delta_n)}{\mathrm{d}\delta_1 \cdots \mathrm{d}\delta_n}\bigg|_{\delta_a=0}$$

How do we evaluate them?

With separate universe simulations



Responses and N-point functions

Power spectrum, Bispectrum, Trispectrum …

$$\left\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\right\rangle = \quad P_m(\boldsymbol{k}) \quad (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')$$

$$\left\langle \delta(\boldsymbol{p}_1)\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\right\rangle = \quad B_m(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{p}_1) \quad (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}'+\boldsymbol{p}_1)$$

$$\left\langle \delta(\boldsymbol{p}_2)\delta(\boldsymbol{p}_1)\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\right\rangle = \quad T_m(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{p}_1,\boldsymbol{p}_2) \quad (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}'+\boldsymbol{p}_1+\boldsymbol{p}_2)$$

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Small scale (hard) modes

Large scale (soft) modes

Responses and N-point functions

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Small scale (hard) modes

Large scale (soft) modes

Modulation of the power spectrum P(**k**) by large-scale modes

i.e. **Responses!**

Responses and N-point functions

Power spectrum, Bispectrum, Trispectrum …

$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle = P_m(\boldsymbol{k}) \quad (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')$$

N+2 squeezed correlations described by the N-th response

$$\langle \delta(\boldsymbol{p}_1)\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle = B_m(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{p}_1) \quad (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}'+\boldsymbol{p}_1)$$

$$\lim_{p_a\to 0}\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)\rangle_{c,\mathcal{R}_n} = n!\,\mathcal{R}_n(k,\text{angles})\,P_m(k)\prod_{a=1}^{n}P_L(p_a)$$

hard

soft

Response

Small scale (hard) modes

Large scale (soft) modes

Modulation of the power spectrum P(**k**) by large-scale modes

i.e. **Responses!**

Squeezed bispectrum example

$$\lim_{p\to 0} \left\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}) \right\rangle_c$$

hard soft

Squeezed bispectrum example

$$2\Big[F_2(\boldsymbol{k},\boldsymbol{p})P_{\rm L}(k)+F_2(\boldsymbol{k}',\boldsymbol{p})P_{\rm L}(k')\Big]P_{\rm L}(p)$$

Result is valid only if all modes are linear

With Standard Perturbation Theory

$$\lim_{p\to 0}\ \langle\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p})\rangle_c$$

hard **soft**

Squeezed bispectrum example

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Result is valid only if all modes are linear

With Standard Perturbation Theory

$$\lim_{p\to 0}\ \langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p})\rangle_c$$

hard **soft**

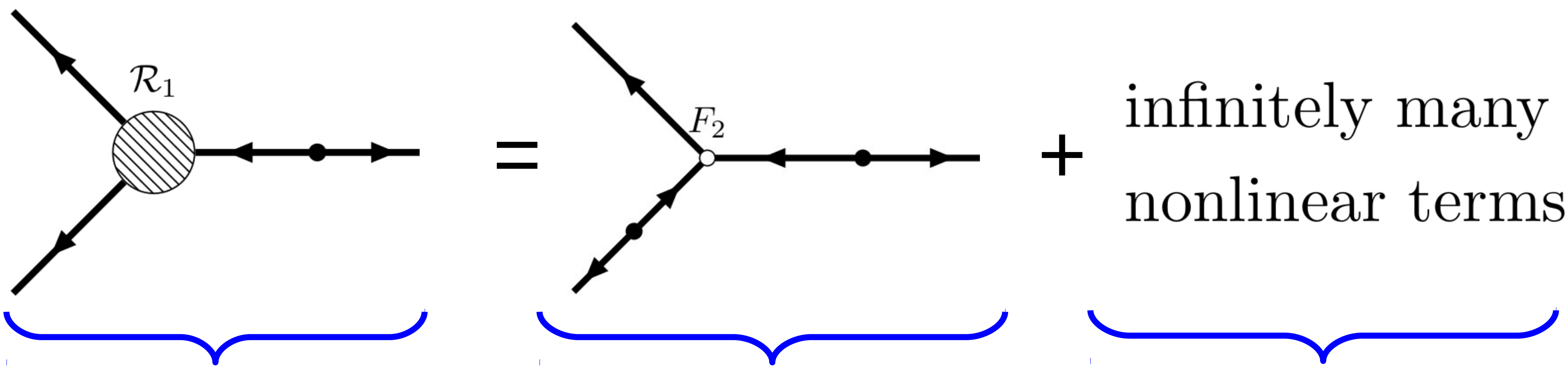
With responses

Result is valid for linear p, ***but any nonlinear k, k' !***

$$\mathcal{R}_1(k,\mu_{\boldsymbol{k},\boldsymbol{p}})P_m(k)P_{\rm L}(p)$$

Squeezed bispectrum example

Responses as an extension of perturbation theory …



Responses are a resummed interaction

Analytical, but insufficient.

Accessible with simulations

but any nonlinear k, k' !

$$\mathcal{R}_1(k, \mu_{\boldsymbol{k},\boldsymbol{p}}) P_m(k) P_{\rm L}(p)$$

Response decomposition

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\, \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

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All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.

Response decomposition

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\, \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

Measure the response to each specific large-scale configuration;

What we will get from simulations.

All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.

Response decomposition

Large-scale overdensity

Large-scale tidal field

$$\mathcal{R}_1 \longrightarrow R_1(k)\delta(\boldsymbol{p}) + R_K(k)\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{p})$$

Response to overdensity

Response to tidal field

Response decomposition

$$
\begin{aligned}
\mathcal{R}_2 \longrightarrow\ & R_1(k)\left[\delta^{(2)}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right] + R_K(k)\left[\hat{k}^i\hat{k}^j K^{(2)}_{ij}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right] \\
+\ & \frac{1}{2}R_2(k)\left[\delta(\boldsymbol{p}_1)\delta(\boldsymbol{p}_2)\right] + R_{K\delta}(k)\left[\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{p}_1)\delta(\boldsymbol{p}_2)\right] \\
+\ & R_{K^2}(k)\left[K_{ij}(\boldsymbol{p}_1)K^{ij}(\boldsymbol{p}_2)\right] + R_{K.K}(k)\left[\hat{k}^i\hat{k}^j K_{il}(\boldsymbol{p}_1)K^l{}_j(\boldsymbol{p}_2)\right] \\
+\ & R_{KK}(k)\left[\hat{k}^i\hat{k}^j\hat{k}^l\hat{k}^m K_{ij}(\boldsymbol{p}_1)K_{lm}(\boldsymbol{p}_2)\right] + R_{\hat{\Pi}}(k)\left[\hat{k}^i\hat{k}^j\hat{\Pi}_{ij}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right]
\end{aligned}
$$

Response coefficients

All 2nd order large-scale operators

Generalizations to any order are always straightforward, just more cumbersome.

Separate universe simulations

Nitty-gritty: Li et al (1401.0385) ; Wagner et al (1409.6294); Schmidt et al (1803.03274);

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\, \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

Response to specific perturbations

All possible configurations of large-scale density/tidal fields;

Separate universe simulations

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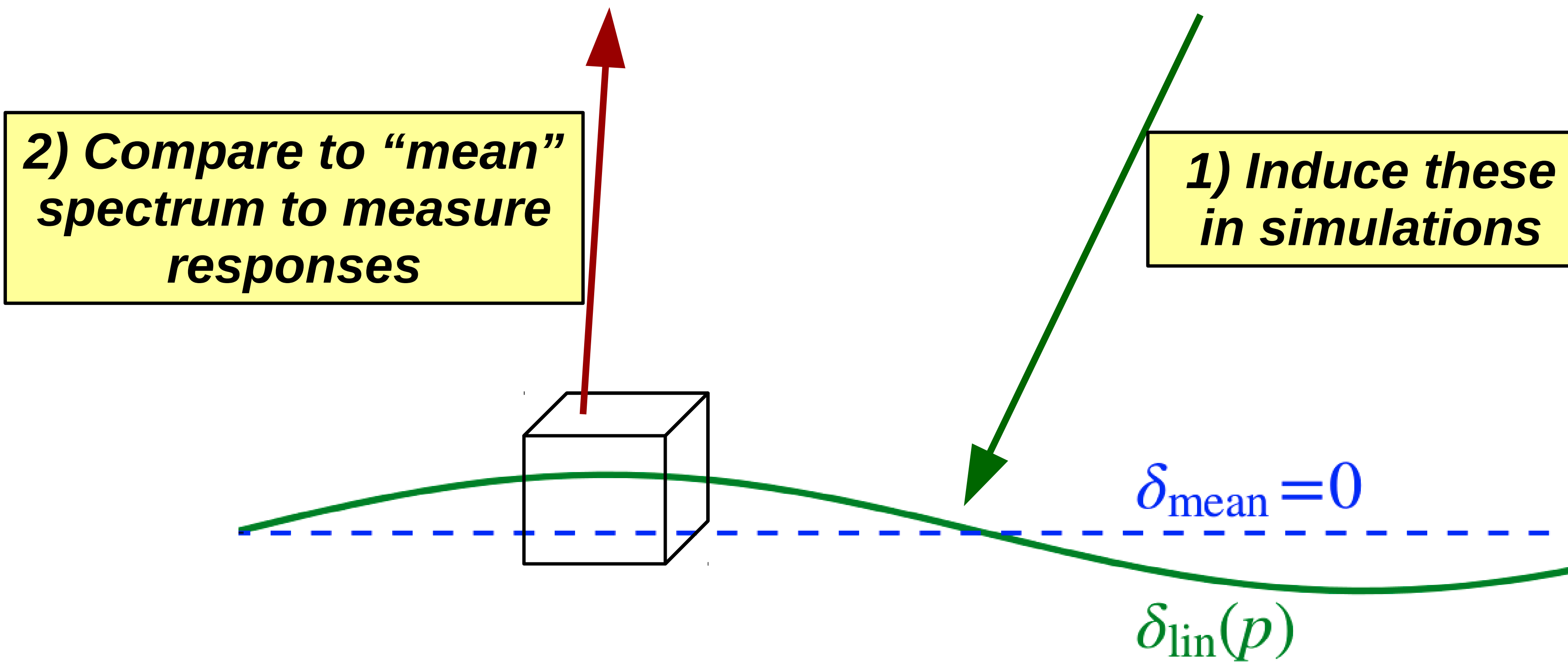


Separate universe simulations

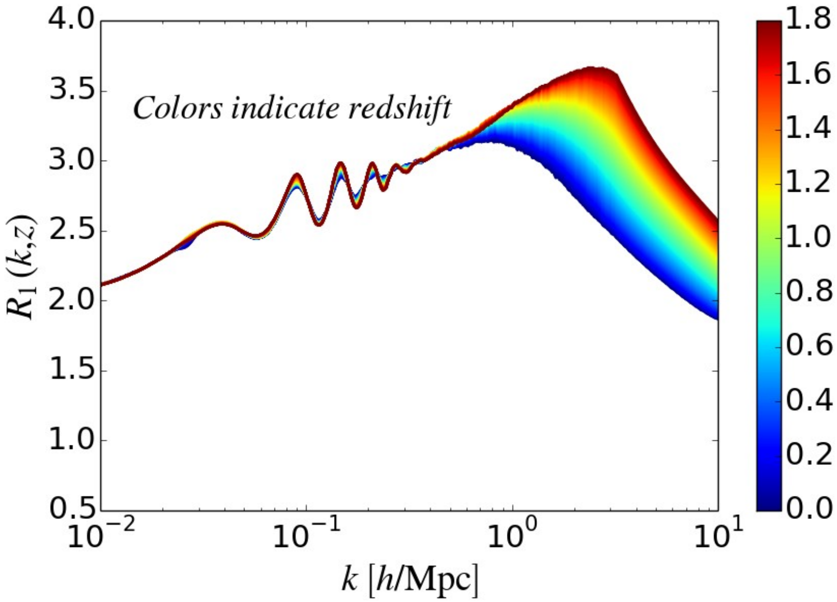
$$P_m(\boldsymbol{k}, \boldsymbol{x}) = P_m(k)\left[1 + R_1(k)\delta(\boldsymbol{x}) + R_K(k)\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{x})\right]$$

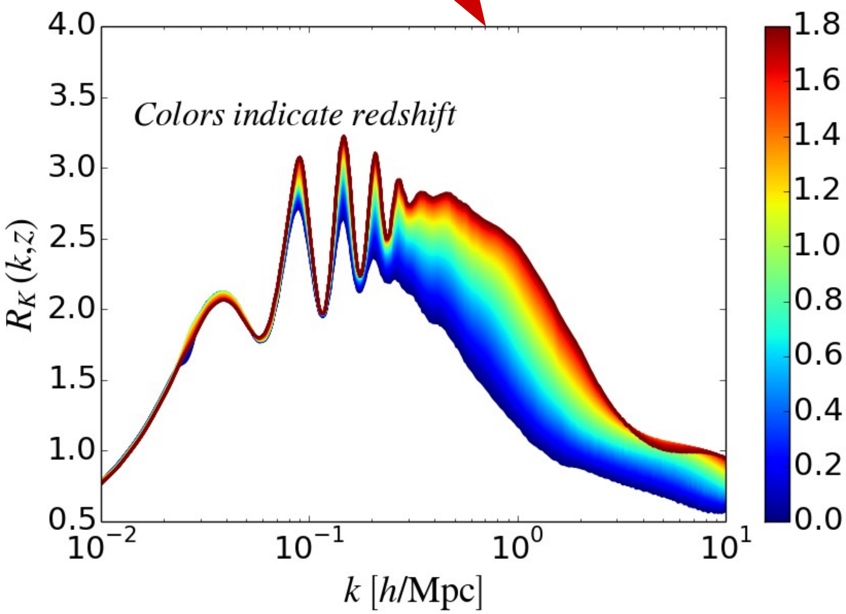
Response to overdensity

Li et al (1401.0385) ; Wagner et al (1409.6294)

Response to tidal field

Schmidt et al (1803.03274)





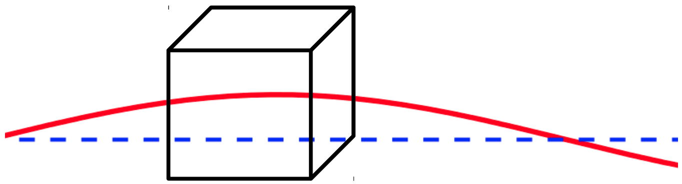
To keep in mind then ...

Responses describe the coupling ***of large-to-small scale modes*** *in the* ***nonlinear regime***

$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)\rangle_{c,\mathcal{R}_n} \propto \mathcal{R}_n(k,\mathrm{angles})\ P_m(k)$$

hard **soft** **response**

Measurable with a few ***Separate Universe simulations.***



Covariances with Responses

Barreira, Schmidt, arXiv:1705.01092
Barreira, Krause, Schmidt, arXiv:1711.07467

3D covariance decomposition

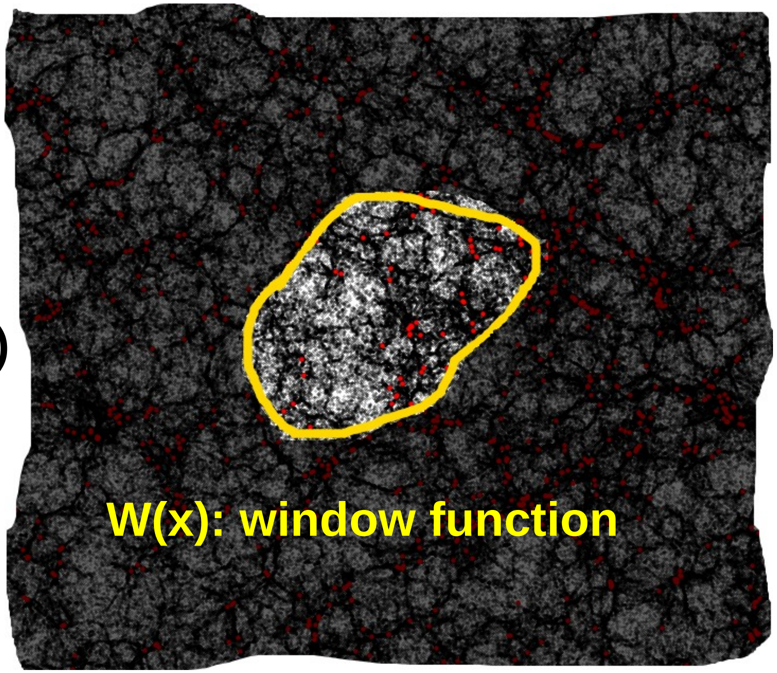
- **Observed, 'windowed' density field**

$$\delta_W(\boldsymbol{x}) = W(\boldsymbol{x})\delta(\boldsymbol{x})$$

- **The power spectrum**

$$\hat{P}_m(\boldsymbol{k}) = \frac{\tilde{\delta}_W(\boldsymbol{k})\tilde{\delta}_W(-\boldsymbol{k})}{V_W}$$

Takada&Hu (1302.6994)



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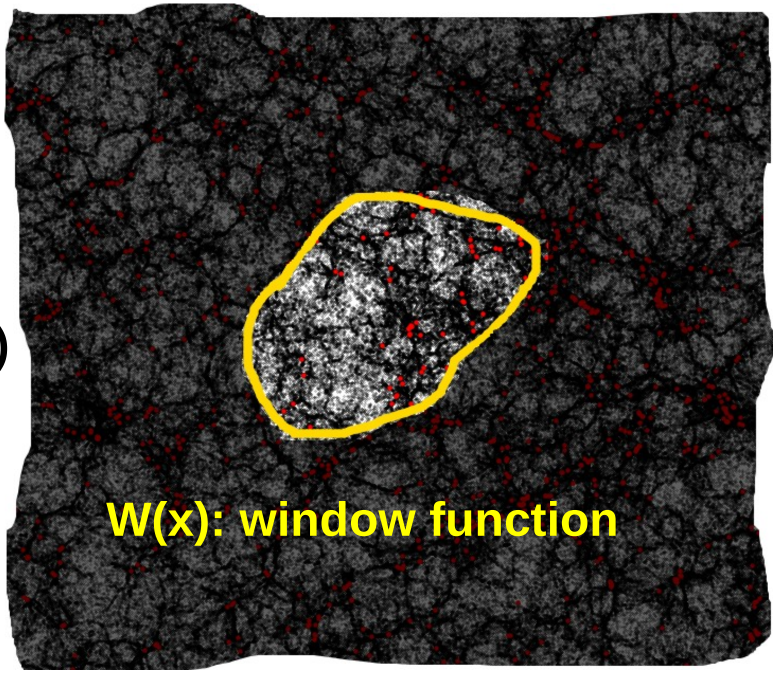

- **The power spectrum covariance**

$$\mathrm{Cov}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \langle \hat{P}_m(\boldsymbol{k}_1)\hat{P}_m(\boldsymbol{k}_2)\rangle - \langle \hat{P}_m(\boldsymbol{k}_1)\rangle\langle \hat{P}_m(\boldsymbol{k}_2)\rangle$$

$$=$$

$$\mathrm{Cov}^{\mathrm{G}}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \mathrm{Cov}^{\mathrm{cNG}}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \mathrm{Cov}^{\mathrm{SSC}}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

Gaussian

Connected non-Gaussian

Super-sample

The Gaussian term : G

- It is the only contribution during the linear regime of structure formation

Trivially given by P(k)

Diagonal

$$\mathrm{Cov}^{\mathrm{G}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{[P_m(k_1)]^2}{V_W}\Big[\delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2) + \delta_D(\boldsymbol{k}_1 - \boldsymbol{k}_2)\Big]$$

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Window function can be included by using the convolved P(k) .

The Gaussian term is well understood !

The Gaussian term : G

• he linear regime of structure formation

Corresponding lensing formulae

Trivially given by P(k)

Assuming Limber's approx., which is okay for l > 20

- **Windowed lensing convergence**

$$\kappa_{\mathcal{W}}(\boldsymbol{\theta}) = \mathcal{W}(\boldsymbol{\theta}) \int \mathrm{d}\chi g(\chi) \delta(\boldsymbol{\theta}, \chi)$$

- **Lensing power spectrum**

$$C(\boldsymbol{\ell}) = \int \mathrm{d}\chi \frac{g(\chi)^2}{\chi^2} P_m(k_{\boldsymbol{\ell}}, \chi) \qquad k_{\boldsymbol{\ell}_i} = \left(\frac{\boldsymbol{\ell}_i + 1/2}{\chi}, 0 \right)$$

- **Gaussian lensing covariance**

$$\mathrm{Cov}^{\mathrm{G}}_{\kappa}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{C(\ell_1)^2}{\Omega_{\mathcal{W}}} \left[\delta_D(\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2) + \delta_D(\boldsymbol{\ell}_1 - \boldsymbol{\ell}_2) \right]$$

$\mathrm{Cov}^{\mathrm{G}}(\boldsymbol{k}_1, \boldsymbol{k}_2$

Window functi
by using the

Connected non-Gaussian term : cNG

- Describes the **coupling of different Fourier modes due to nonlinear structure formation** .

Parallelogram trispectrum

$$\mathrm{Cov}^{\mathrm{cNG}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{V_W} T_m^{\mathrm{cNG}}(\boldsymbol{k}_1, -\boldsymbol{k}_1, \boldsymbol{k}_2, -\boldsymbol{k}_2)$$

Connected non-Gaussian term : cNG

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- **Extend to the nonlinear regime** with responses if k1 >> k2:

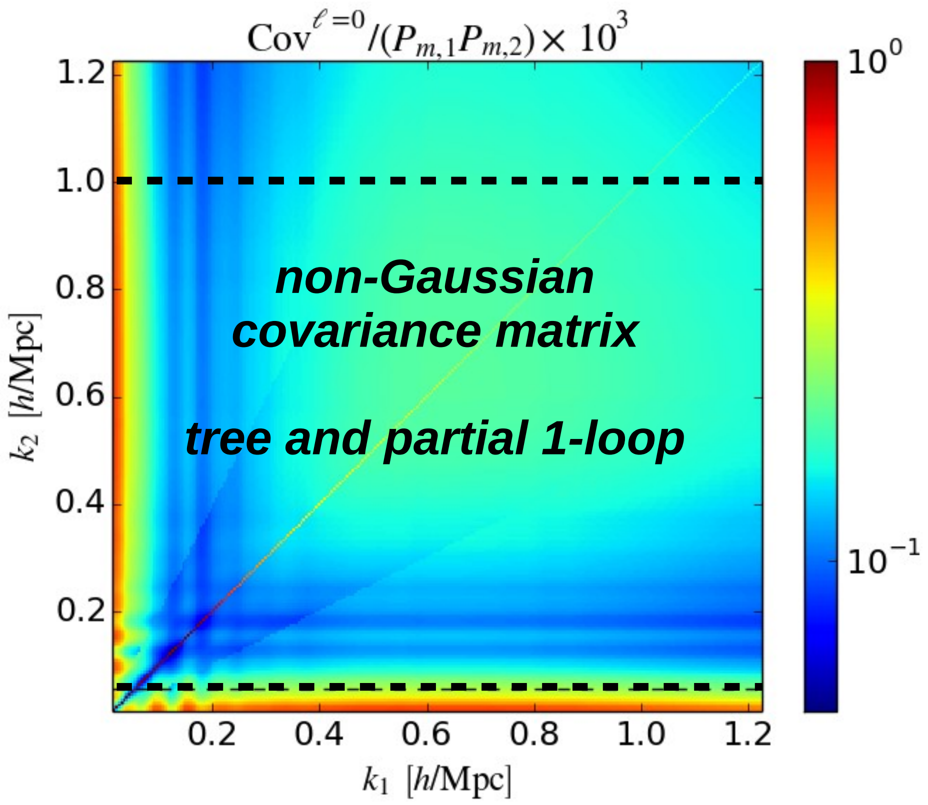
Valid for any nonlinear value of k1 !

hard soft

$$T_m^{\mathrm{cNG}}(\boldsymbol{k}_1, -\boldsymbol{k}_1, \boldsymbol{k}_2, -\boldsymbol{k}_2) = 2\mathcal{R}_2(k_1, \mu_{12}) P_m(k_1) [P_L(k_2)]^2$$

response

cNG : response vs simulations



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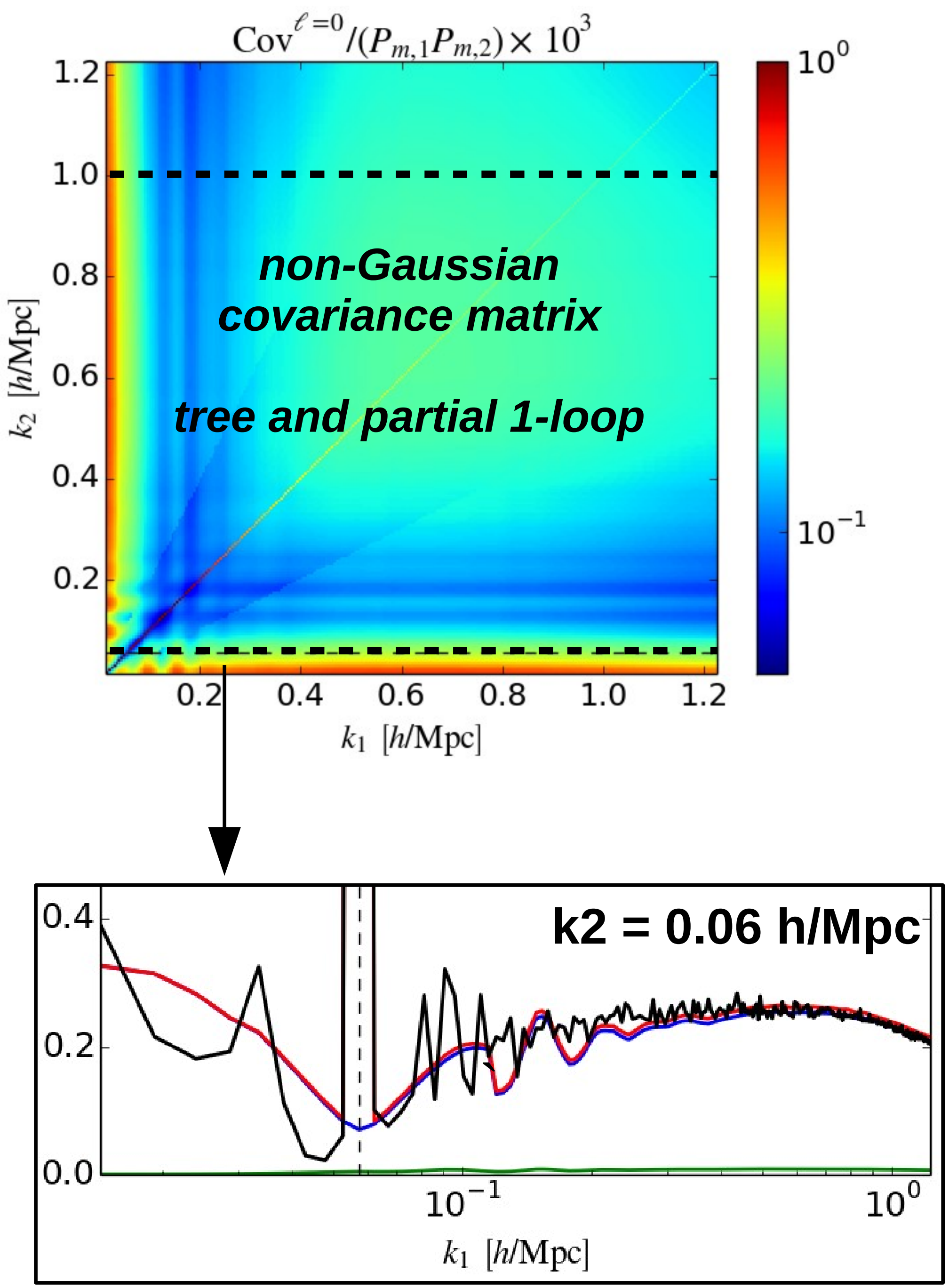
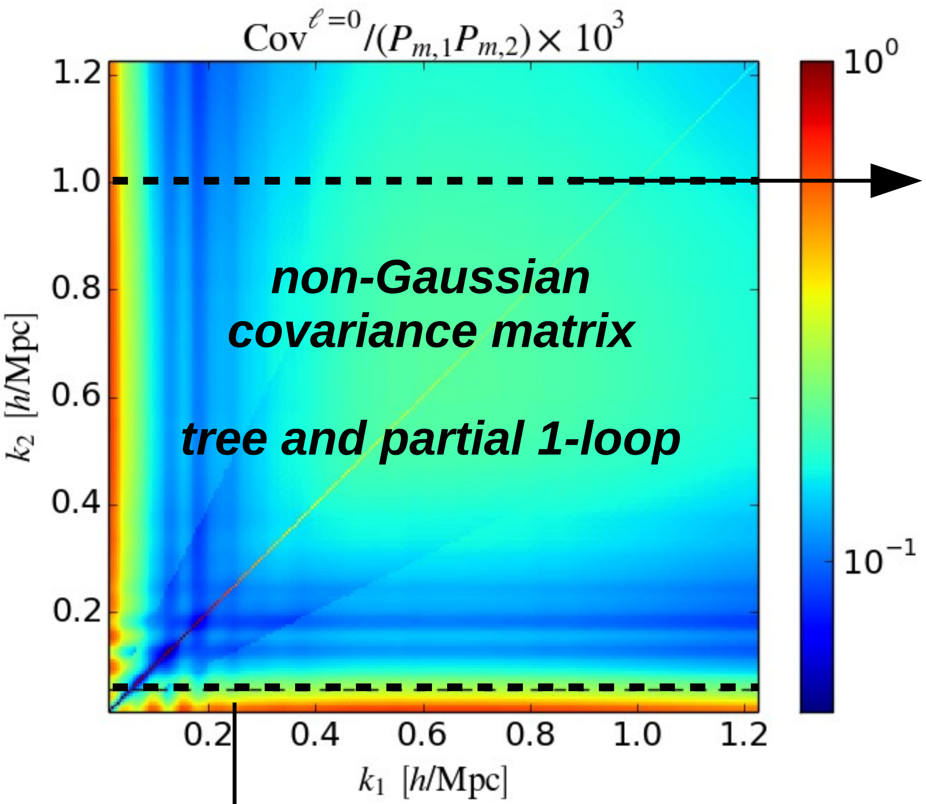
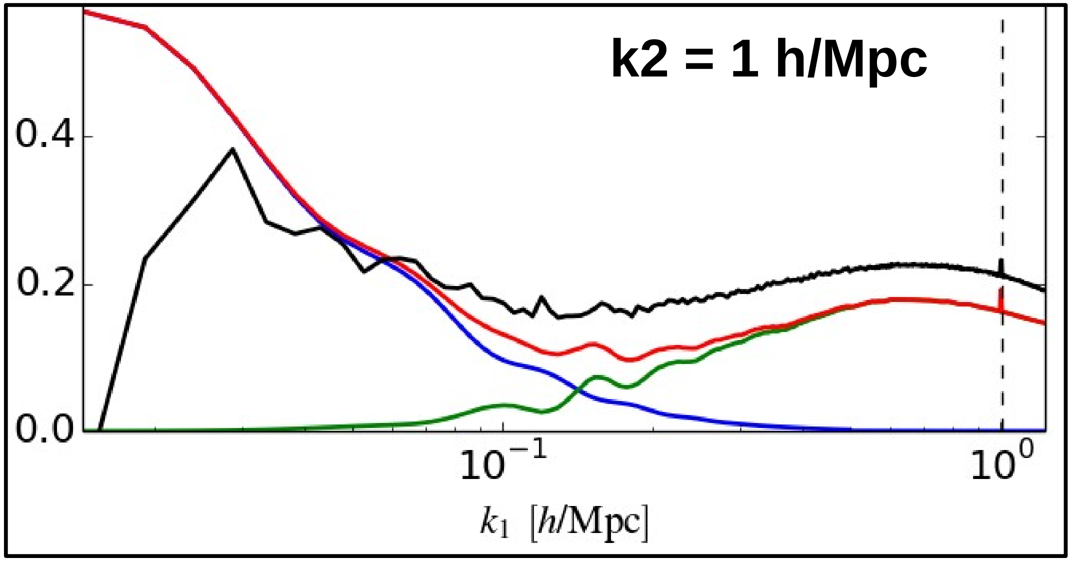


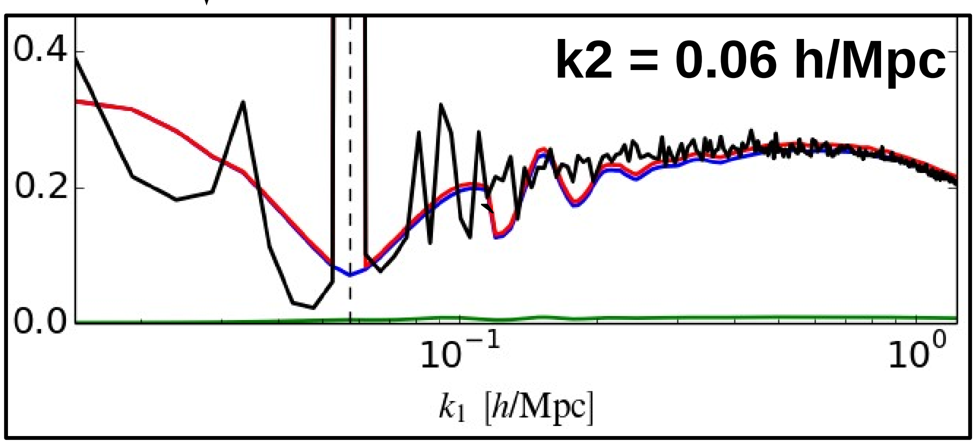
Black : Blot + (2015); over 12000 sims.
Red : response

If one mode is linear : responses capture all there is

cNG : response vs simulations







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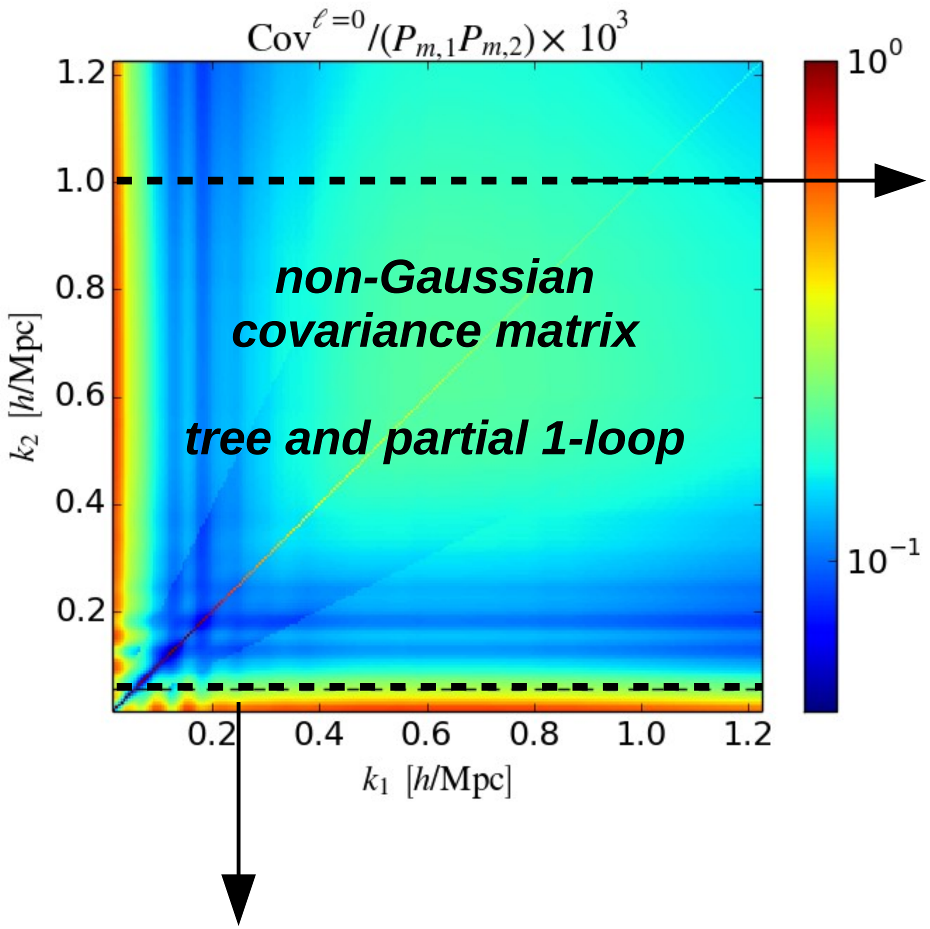
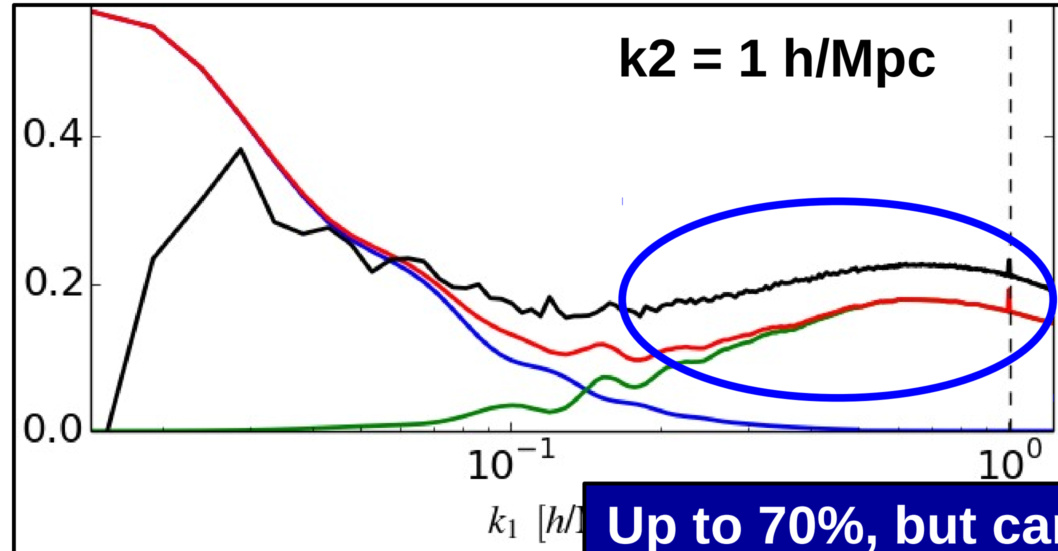
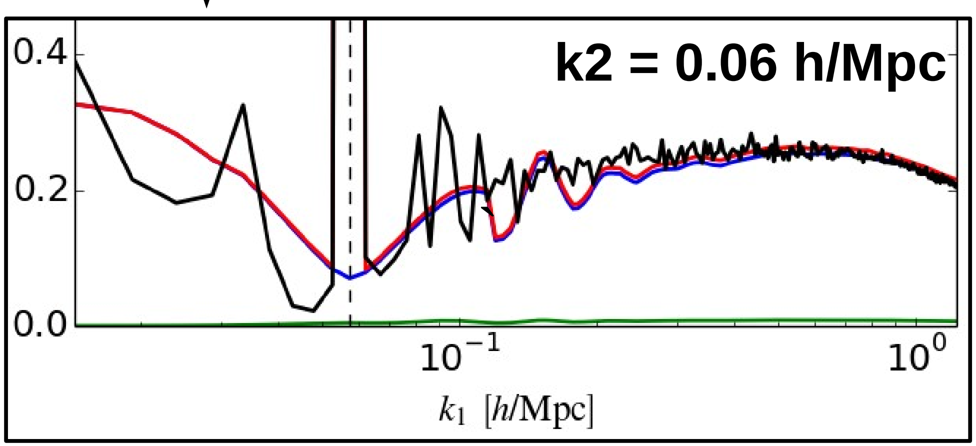




Up to 70%, but can be improved.



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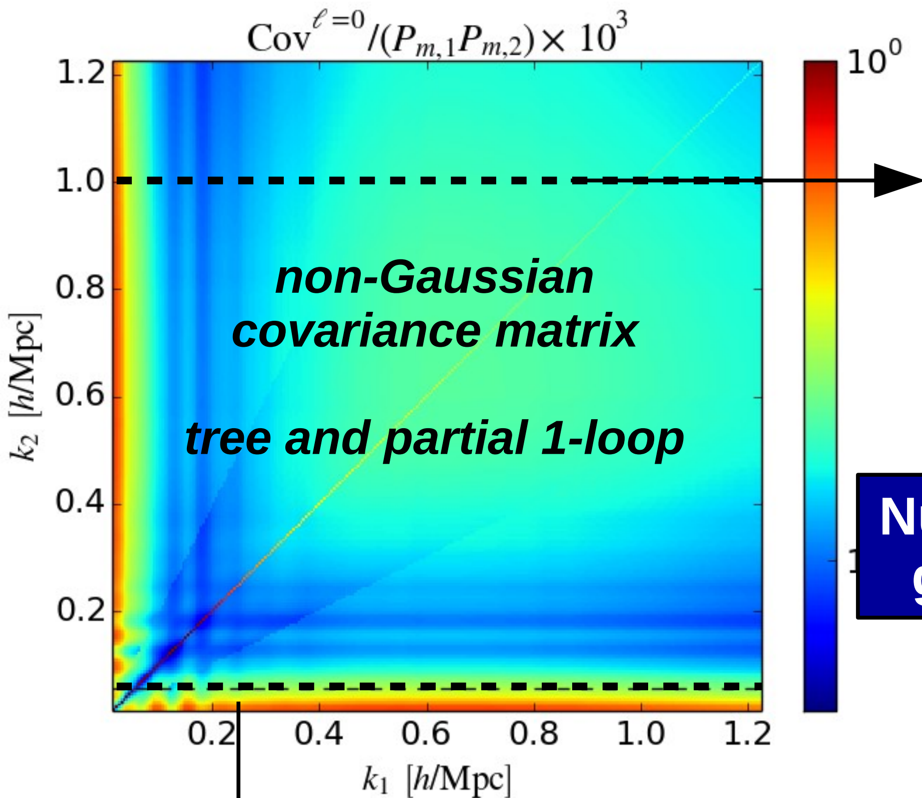
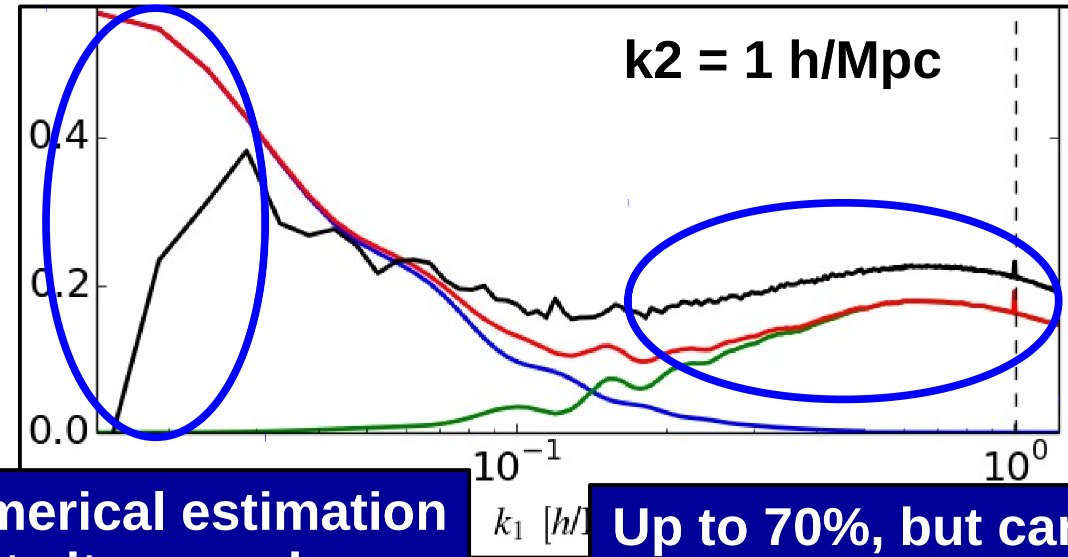
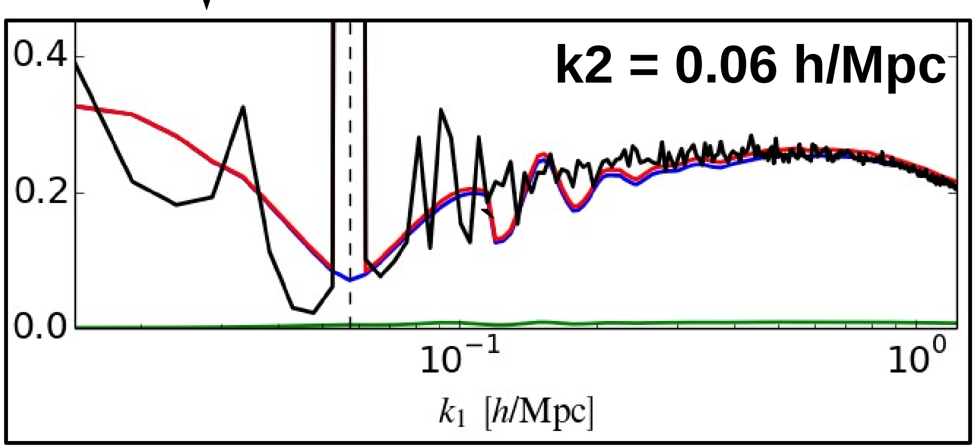




Numerical estimation gets it wrong here.

Up to 70%, but can be improved.



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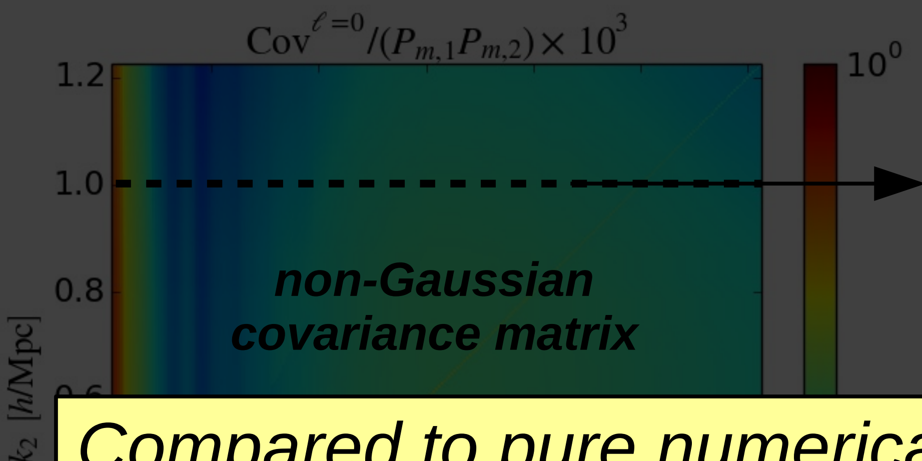
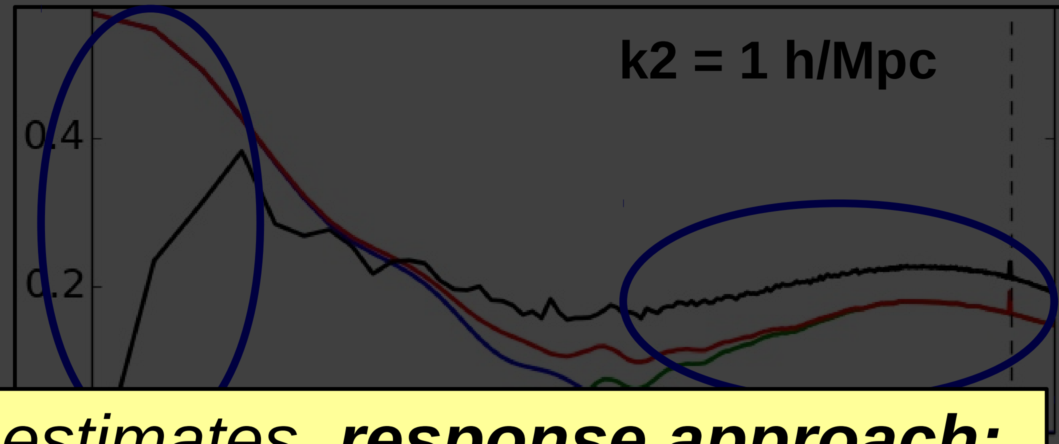
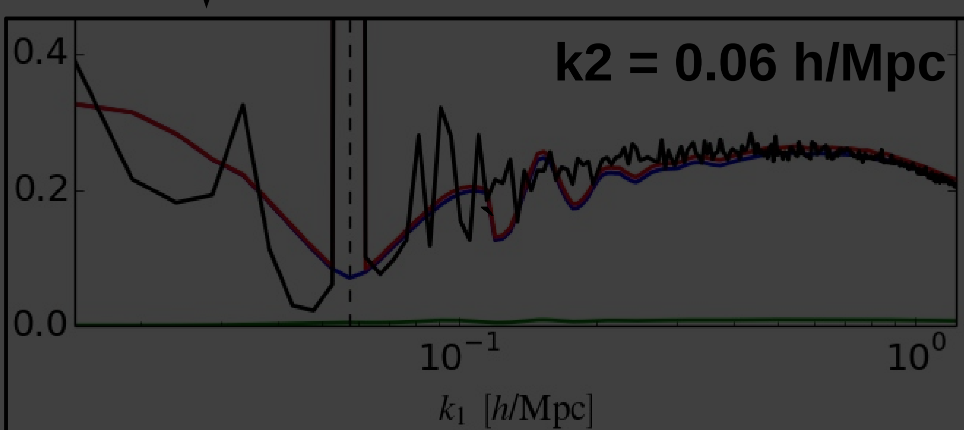




Compared to pure numerical estimates, ***response approach:***

1) Requires ***negligible numerical resources***

2) Is virtually ***noise-free.***



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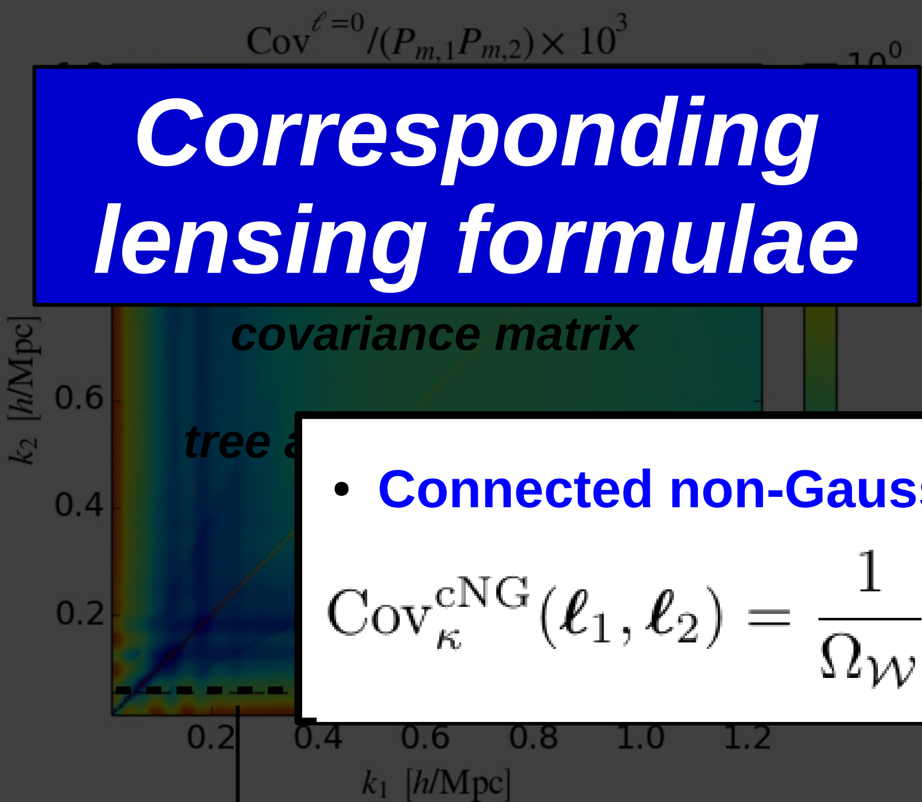
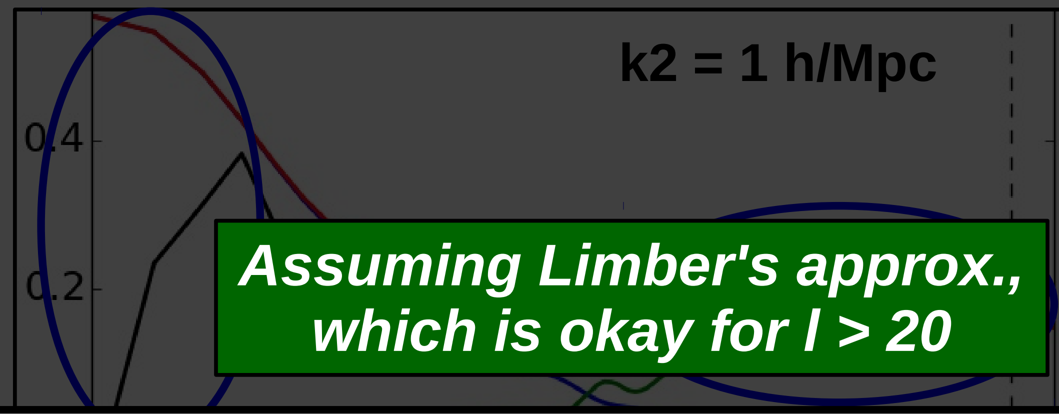
cNG : response vs simulations





Corresponding lensing formulae

Assuming Limber's approx., which is okay for l > 20

- **Connected non-Gaussian lensing covariance**

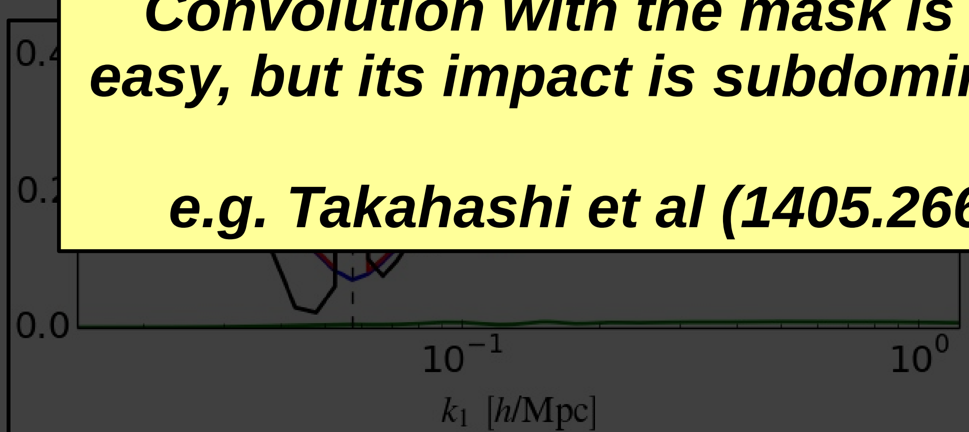
$$\mathrm{Cov}_\kappa^{\mathrm{cNG}}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{1}{\Omega_\mathcal{W}} \int \mathrm{d}\chi \frac{g(\chi)^4}{\chi^6} T_m^{\mathrm{cNG}}(k_{\boldsymbol{\ell}_1}, -k_{\boldsymbol{\ell}_1}, k_{\boldsymbol{\ell}_2}, -k_{\boldsymbol{\ell}_2}, \chi)$$

$$k_{\boldsymbol{\ell}_i} = \left(\frac{\boldsymbol{\ell}_i + 1/2}{\chi}, 0\right)$$

Black : Blot + (201
d : response

Convolution with the mask is not easy, but its impact is subdominant !

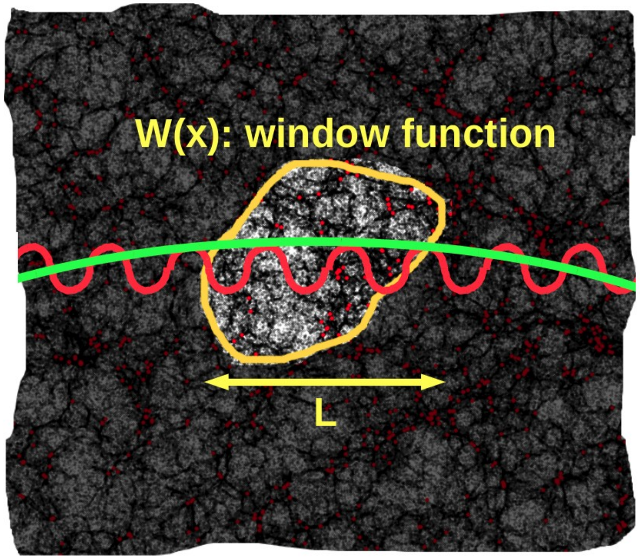
e.g. Takahashi et al (1405.2666)



If one mode is linear : responses capture all there is

The super-sample term : SSC

- Describes the coupling of modes inside the survey with **<u>unobserved modes outside the survey</u>**.
- Given by trispectrum terms that get excited by **finiteness of the window function**



$$\mathrm{Cov}^{\mathrm{SSC}}(\boldsymbol{k}_1, \boldsymbol{k}_2) \; = \; \frac{P_m(k_1)P_m(k_2)}{V_W^2} \int_{\boldsymbol{p}} |\tilde{W}(\boldsymbol{p})|^2 \mathcal{R}_1(k_1, \mu_{\boldsymbol{k}_1,\boldsymbol{p}}) \mathcal{R}_1(k_2, \mu_{\boldsymbol{k}_2,\boldsymbol{p}}) P_{\mathrm{L}}(p)$$

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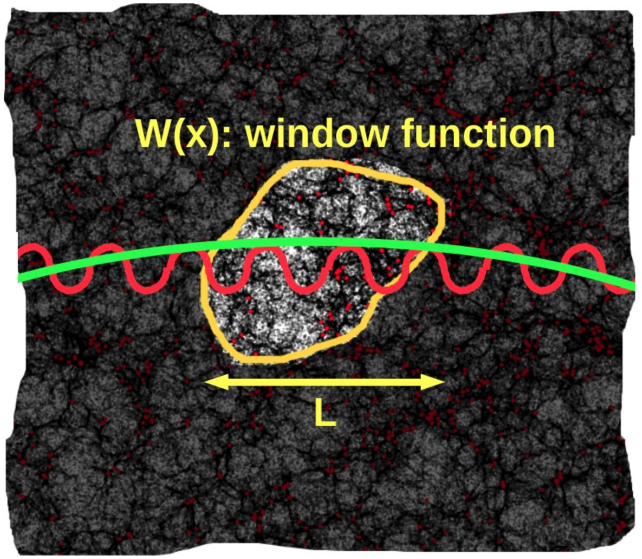

Window function is a low-pass filter that selects $p < \frac{1}{V_W^{1/3}}$

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Super-sample interactions are response interactions

Responses capture SSC completely !

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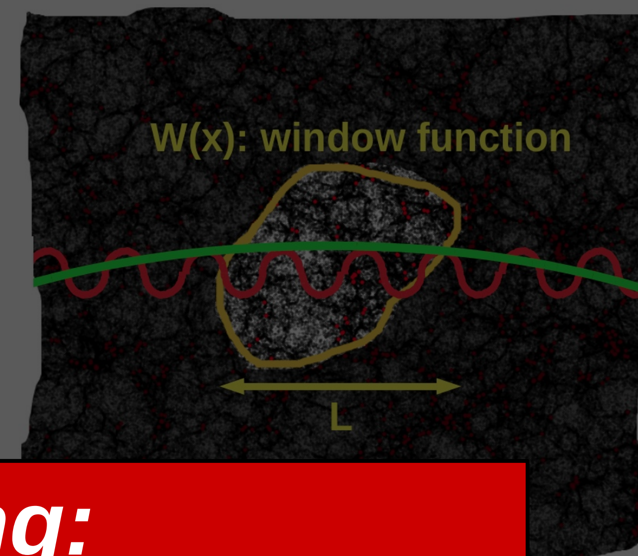
Corresponding lensing formulae

- … et excited by **finiteness of the window function**



Window function is a low…

filter that selects $p <$ …

Warning:
Validity of Limber's approximation at stake because of the long-mode !

$$\mathrm{Cov}^{\mathrm{SSC}}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{P_m(k_1)P_m(k_2)}{V_W^2}\int_{\boldsymbol{p}} |\tilde{W}(\boldsymbol{p})|^2 \mathcal{R}_1(k_1,\mu_{\boldsymbol{k}_1,\boldsymbol{p}})\mathcal{R}_1(k_2,\mu_{\boldsymbol{k}_2,\boldsymbol{p}}) P_{\mathrm{L}}(p)$$

Super-sample interactions are response interactions

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SSC beyond flat-sky/Limber's approx

$$C(\boldsymbol{\ell}) = \int \mathrm{d}\chi \frac{g(\chi)^2}{\chi^2} P_m(k_{\boldsymbol{\ell}}, \chi)$$

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Account for long modes with responses

$$C(\boldsymbol{\ell}, \hat{\boldsymbol{n}}|\boldsymbol{p}) = \int \mathrm{d}\chi \frac{g(\chi)^2}{\chi^2} P_m(k_{\boldsymbol{\ell}}) \left(1 + \int_{\boldsymbol{p}} \left[R_1(k_\ell)\delta(\boldsymbol{p}) + R_K(k_\ell)\hat{k}^i_{\boldsymbol{\ell}}\hat{k}^j_{\boldsymbol{\ell}} K_{ij}(\boldsymbol{p})\right] e^{i\chi\boldsymbol{p}\hat{\boldsymbol{n}}}\right)$$

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- **Never assuming Limber for the long-mode;**

$$\mathrm{Cov}^{\mathrm{SSC}}(\ell_1, \ell_2) = \frac{1}{\Omega_W^2} \sum_{LM} |b_{LM}|^2 \sigma^L_{\ell 1, \ell 2}$$

Power spectrum of mask on the curved sky

Variance-like integral that accounts for 3D long-mode.

SSC beyond flat-sky/Limber's approx

$$C(\boldsymbol{\ell}) = \int \mathrm{d}\chi \frac{g(\chi)^2}{\chi^2} P_m(k_{\boldsymbol{\ell}}, \chi)$$

Limber's approximation underestimates SSC matrix elements by ~10% for f_sky ~ 0.3-0.4 !

$$C(\boldsymbol{\ell}, \hat{\boldsymbol{n}}|\boldsymbol{p}) = \int \mathrm{d}\chi \frac{g(\chi)^2}{\chi^2} P_m(k_\ell) \left(1 + \int_{\boldsymbol{p}} \left[R_1(k_\ell)\delta(\boldsymbol{p}) + R_K(k_\ell)\hat{k}^i_{\boldsymbol{\ell}}\hat{k}^j_{\boldsymbol{\ell}} K_{ij}(\boldsymbol{p})\right] e^{i\chi \boldsymbol{p}\hat{\boldsymbol{n}}}\right)$$

- **Never assuming Limber for the long-mode;**

Don't forget responses to tidal fields, if you want SSC entries to better than 5% !

$$\mathrm{Cov}^{SSC}(\ell_1, \ell_2) = \frac{1}{\Omega_W^2} \sum_{LM} |b_{LM}|^2 \sigma^L_{\ell 1, \ell 2}$$

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Lensing covariance summary

$$\mathrm{Cov}_\kappa(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \mathrm{Cov}_\kappa^{\mathrm{G}} + \mathrm{Cov}_\kappa^{\mathrm{cNG}} + \mathrm{Cov}_\kappa^{\mathrm{SSC}}$$

Lensing covariance summary

Solved !

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Responses capture most of it ,

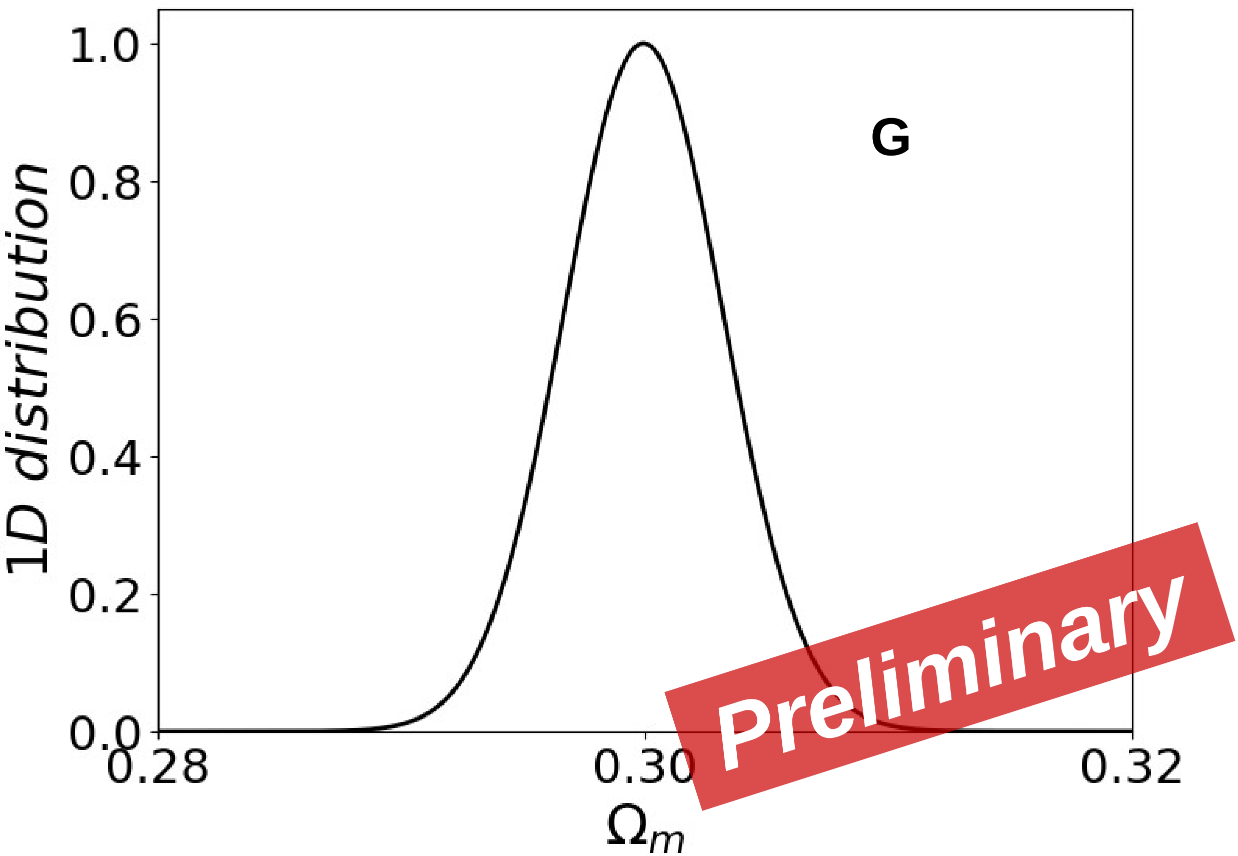
<u>but do we really need it ?</u>

Forecast covariance requirements

Euclid-like lensing setup

- 3 tomographic bins
- 20 ell bins in [20, 5000]
- Mask: spherical cap 15000 deg^2
- Source density: 30 / arcmin^2

w/ CosmoLike , Krause&Eifler (1601.05779)



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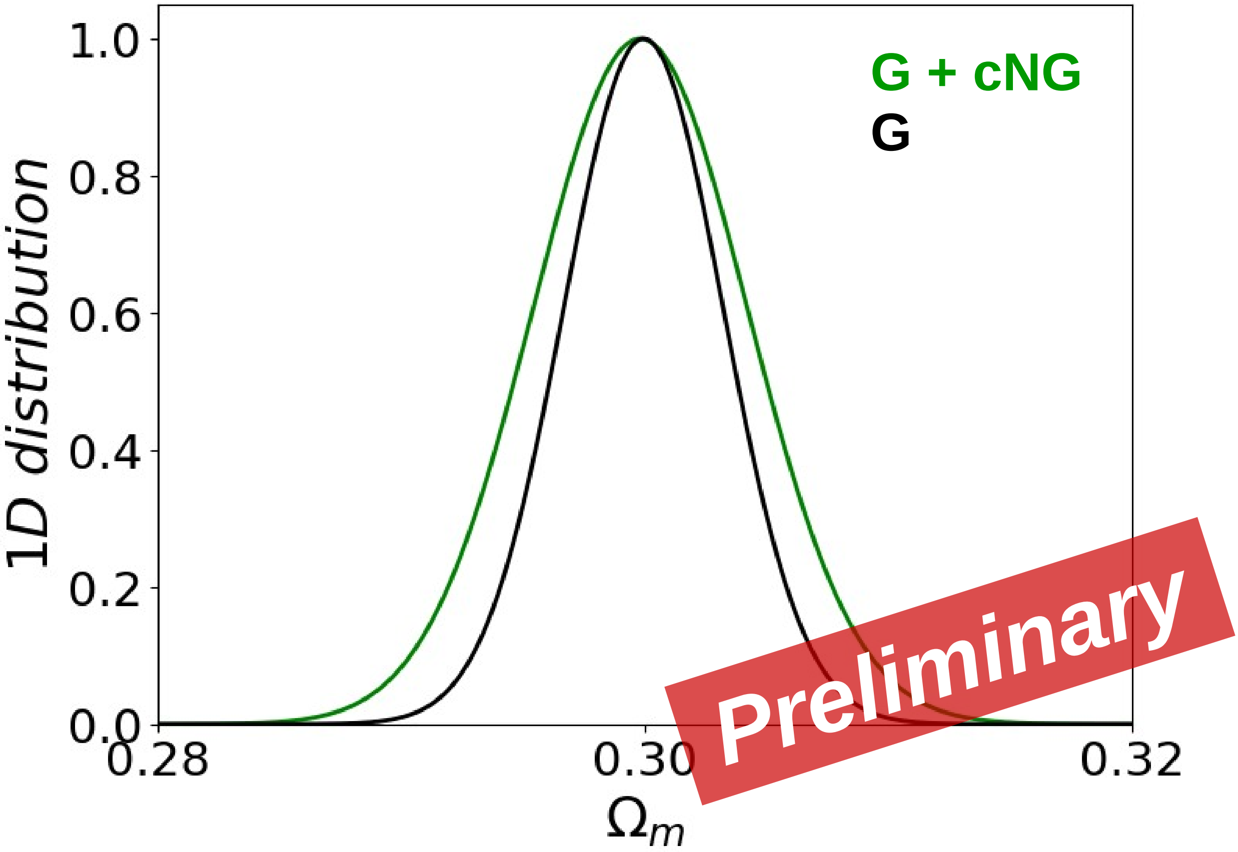
Relative to G, cNG increases error by 34% .



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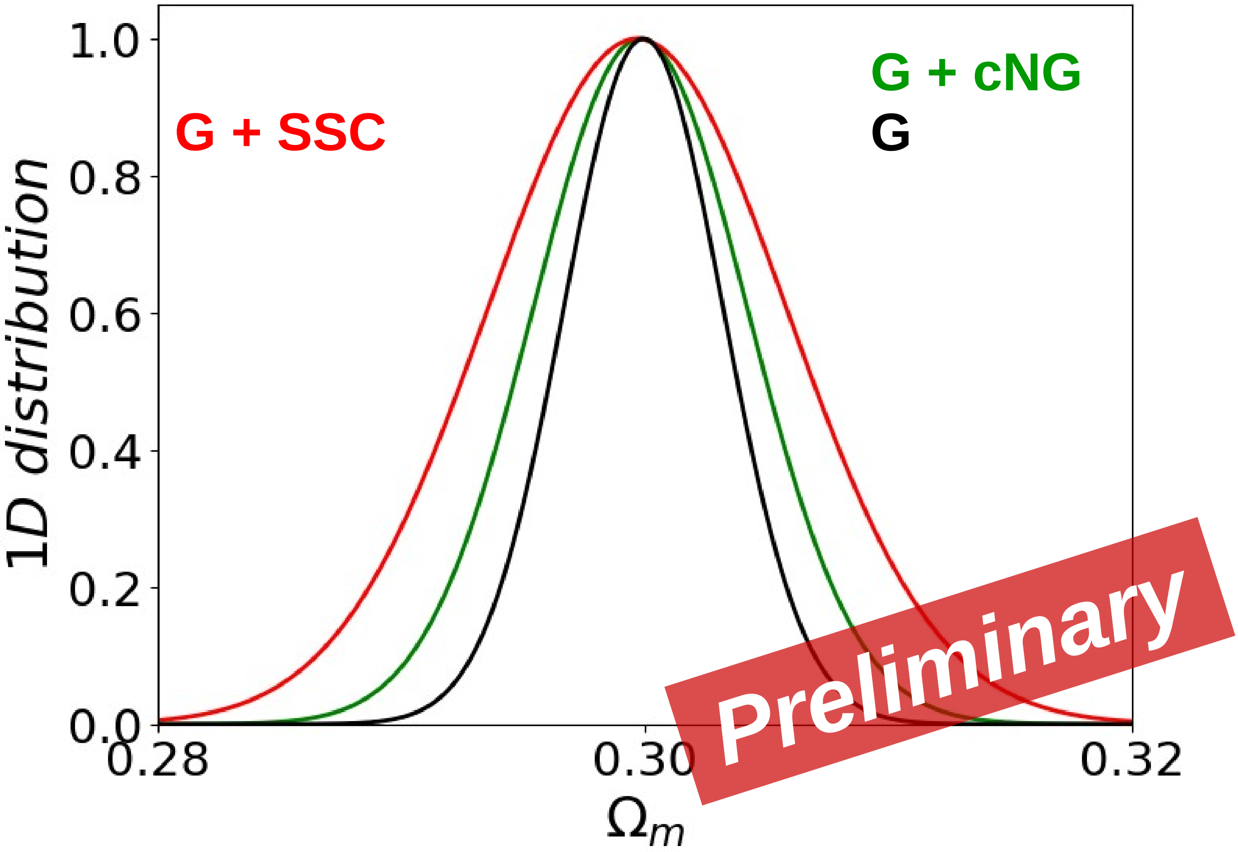

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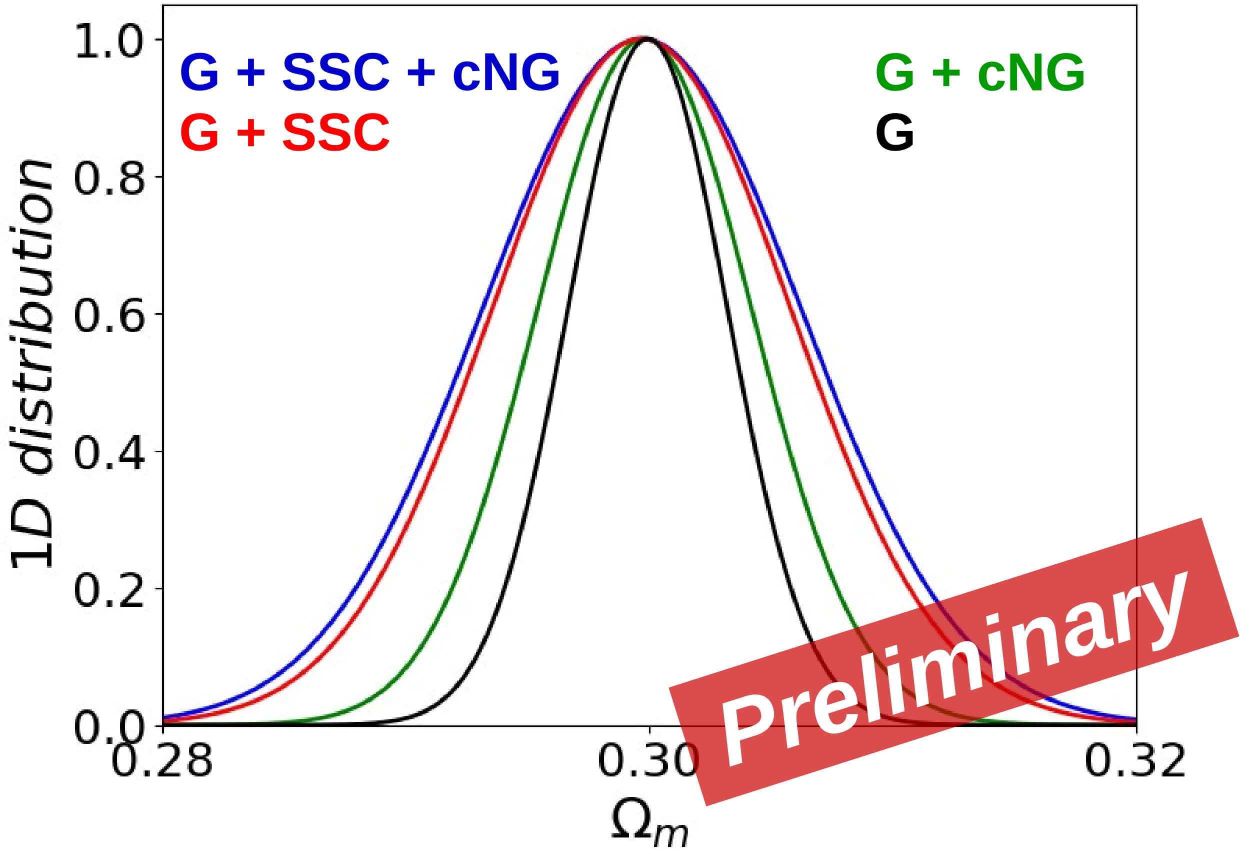
Relative to G, cNG increases error by 34% .

Relative to G+SSC, cNG increases error by only 5% .



Forecast covariance requirements

Euclid-like lensing setup

w/ CosmoLike , Krause&Eifler (1601.05779)

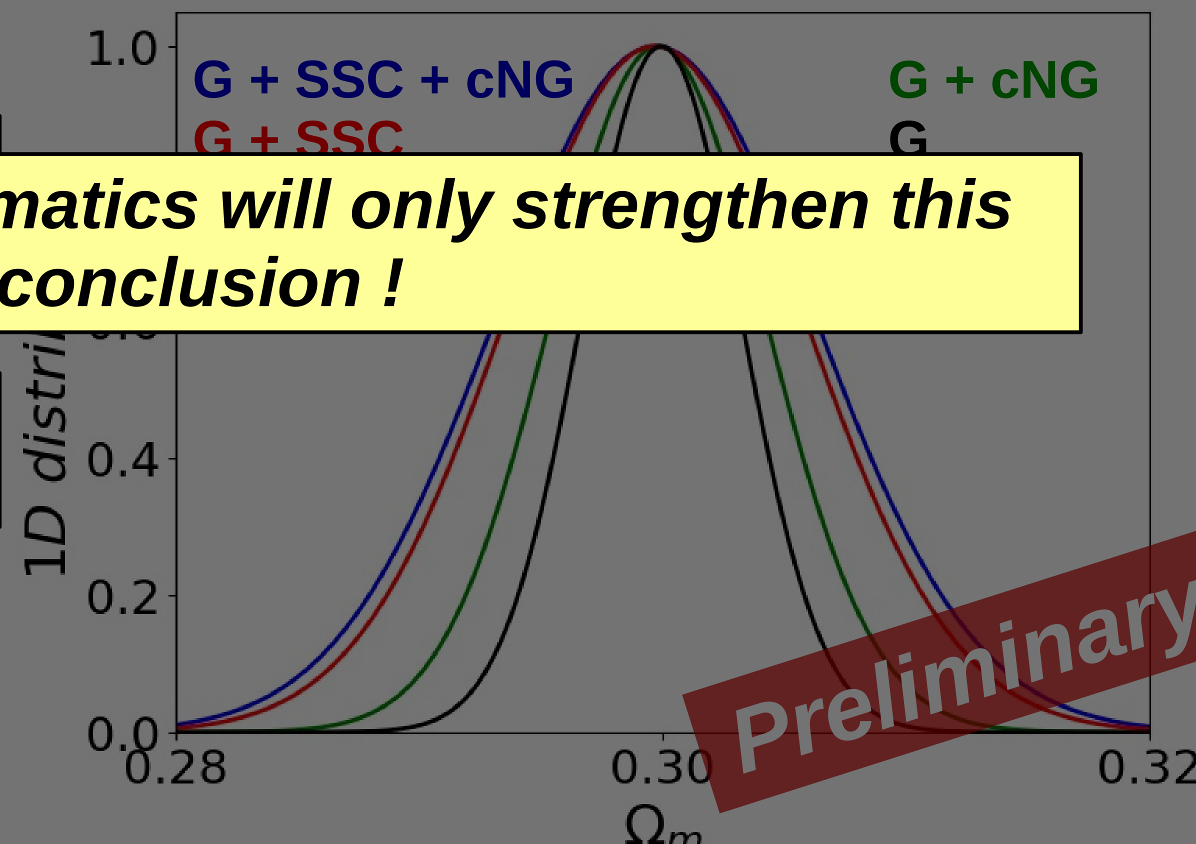
- 3 tomographic bins
- 20 ell bins in [20, 5000]
- Ma
- Sc

In the presence of the dominant off-diagonal SSC term, cNG becomes irrelevant ...

Relat

… including systematics will only strengthen this conclusion !

Relative to G+SSC, cNG increases error by only 5% .



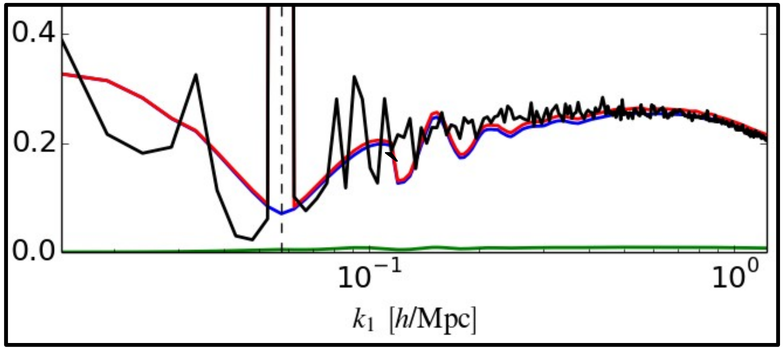
Responses on Sample Covariance

Off-diagonal covariance is ***dominated by responses*** *.*

Solved ! — **Most of it, but small anyway !** — **Solved !**

$$\mathrm{Cov}_\kappa(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \mathrm{Cov}_\kappa^{\mathrm{G}} + \mathrm{Cov}_\kappa^{\mathrm{cNG}} + \mathrm{Cov}_\kappa^{\mathrm{SSC}}$$

***Accurate** covariances with* ***modest numerical resources** !*



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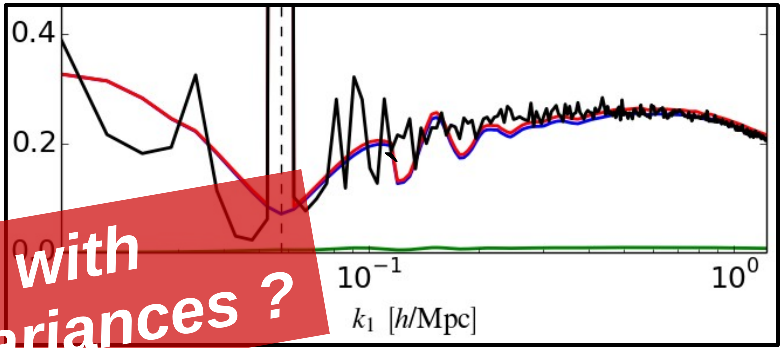

Can't we then live with analytical sample covariances ?

- **Implementation of lensing covariance exists (stay tuned);**
- **Applications to galaxy and cross covariance are possible;**
- **Applications are not limited to power spectra covariances.**