Beyond the power spectrum with large-deviation theory

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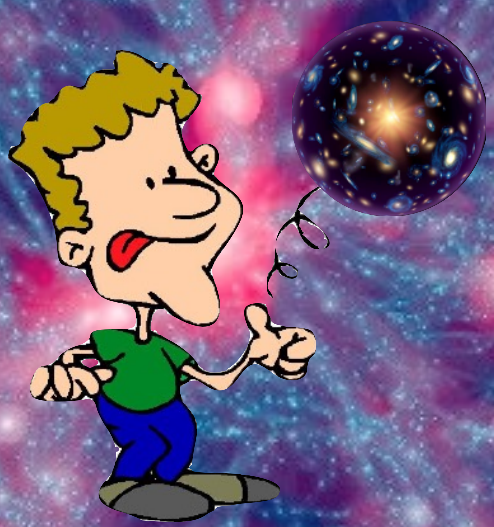
in collaboration with: Cora Uhlemann, Francis Bernardeau, Christophe Pichon

horizon-AGN

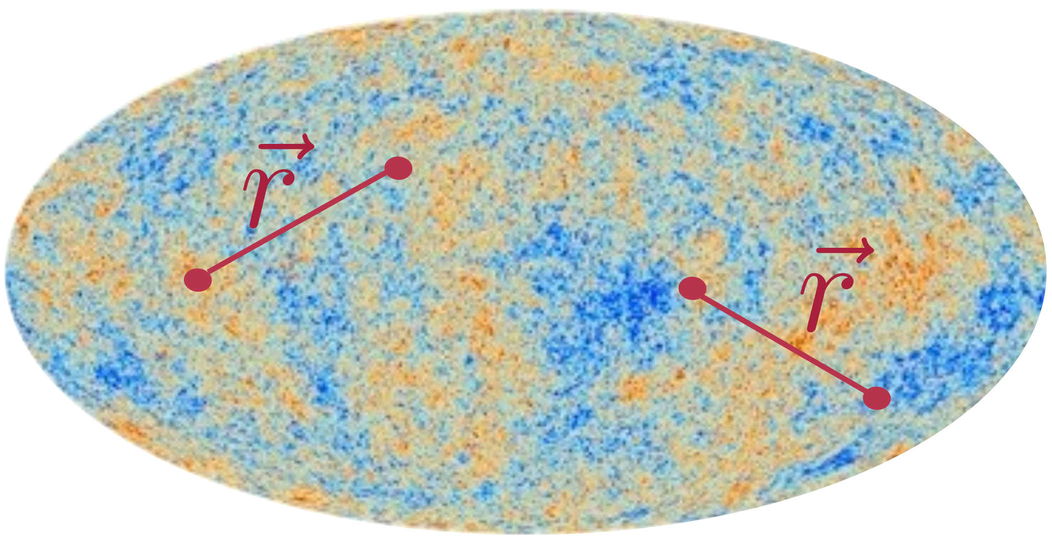


18/04/2018

SCLSS workshop, Oxford

How is the cosmic web woven?

Gaussian primordial fluctuations

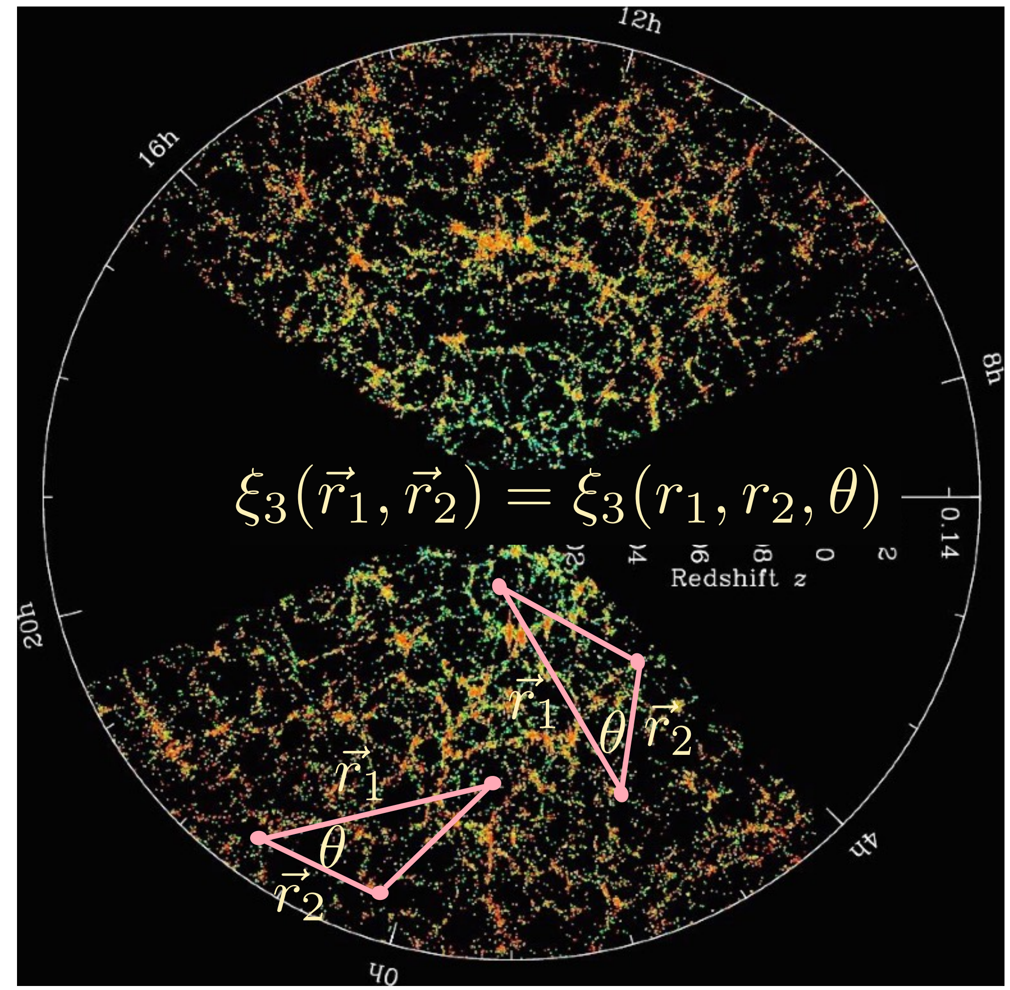


$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle = \xi_2(||\vec{r}||)$$

gravity

expansion

cosmic web: voids, walls, filaments, nodes



Initial state fully described by the 2-pt correlation function (= power spectrum)

Subsequent gravitational evolution is **non-Gaussian**: need to go beyond 2-pt and study **higher order statistics** e.g 3-pt correlation function (= bispectrum)

How to extract information?

Which observables to use?

Power spectrum

Which observables to use?

Power spectrum

+

bispectrum?

Which observables to use?

Power spectrum
+
bispectrum?

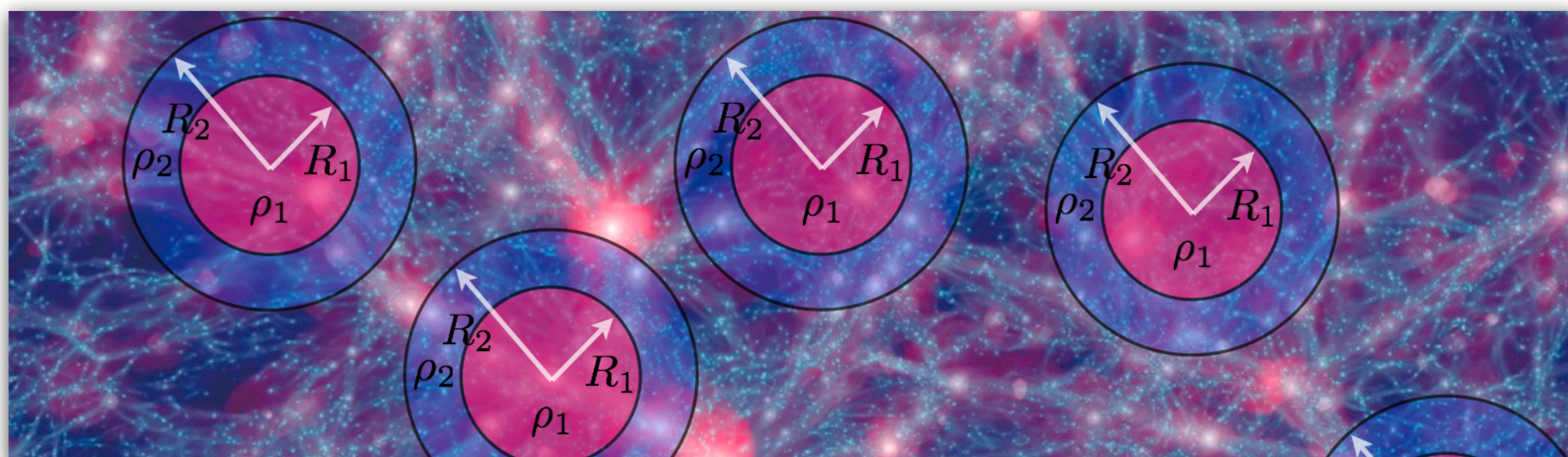
Issues: to enter into the non-linear regime, one probably needs to introduce plenty of free parameters.

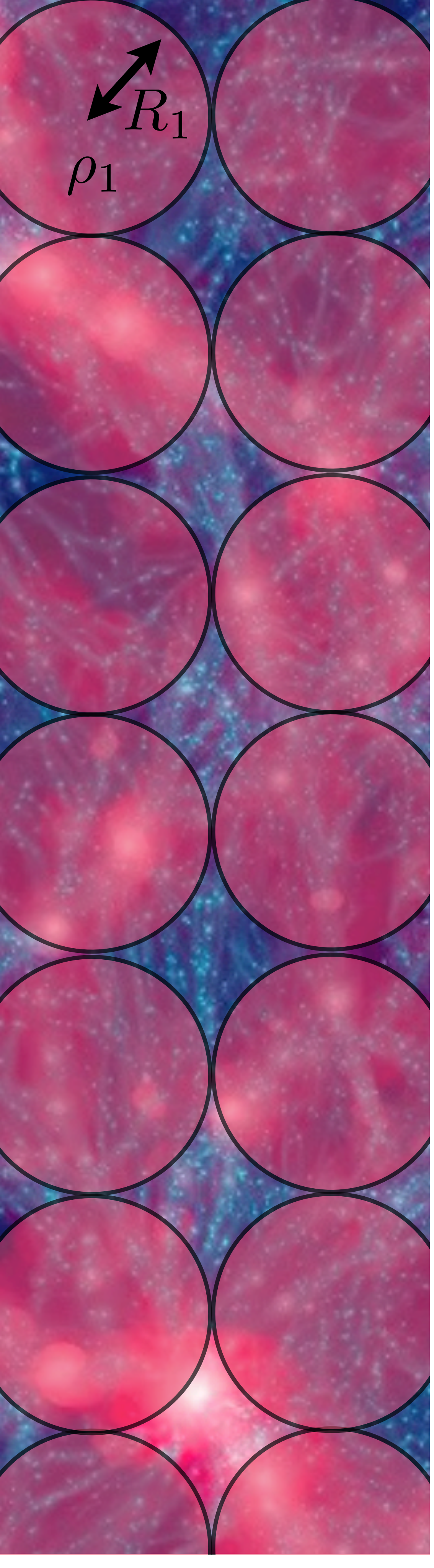
Is it worth it?

Can we find other observables which can be predicted from first principles and can probe the mildly non-linear regime?

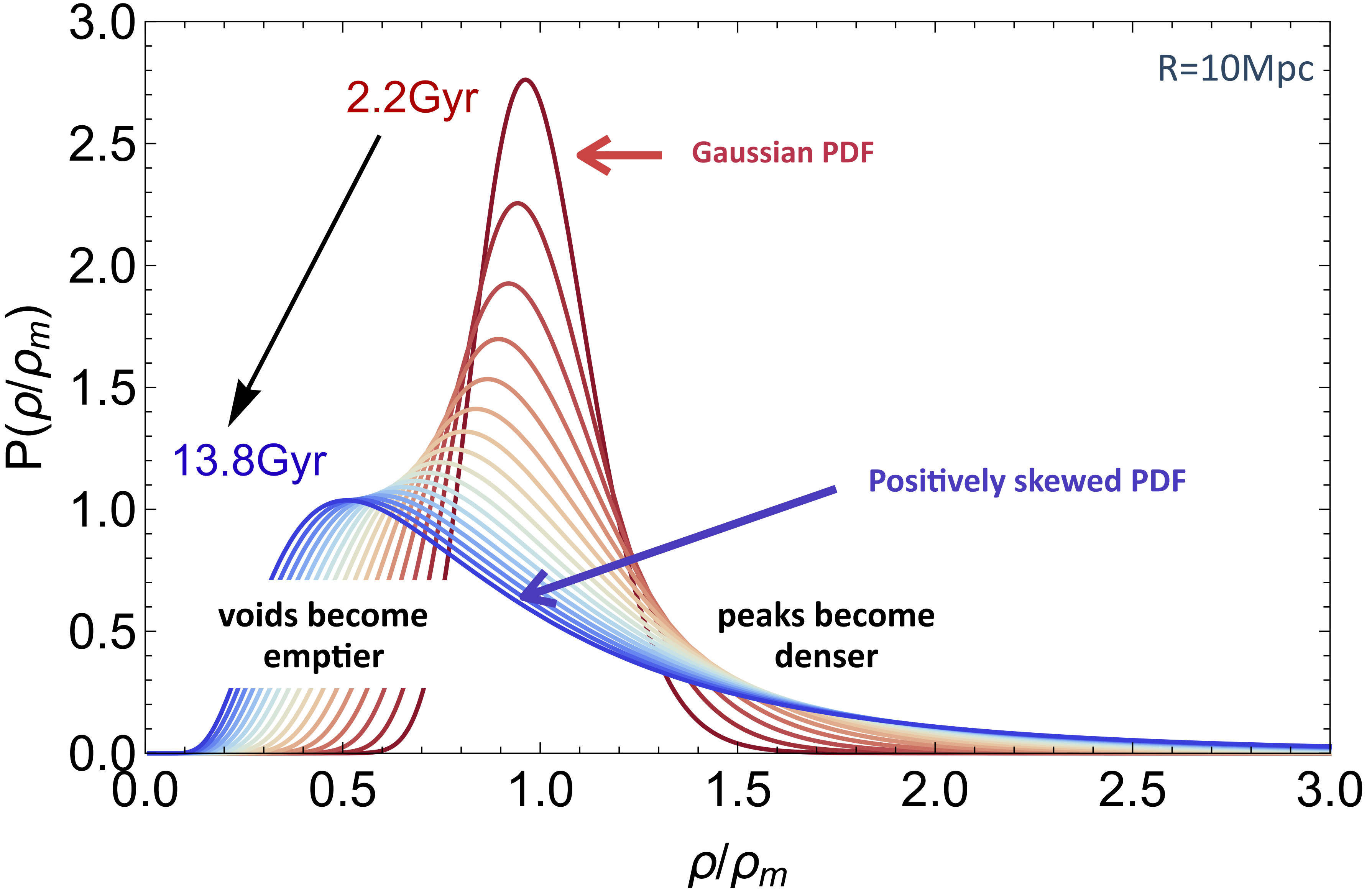
Yes, there is one such configuration: counts in cells, for which the spherical symmetry allows to reduce the effect of the small scales.

$$\mathcal{P}(\rho_1, \cdots, \rho_n) = ?$$





Cosmic density PDF



Can we predict this non-linear density PDF?

From cumulants to PDF

PT can predict the n-th order cumulants whose ratios

$$S_n = \frac{\langle \delta^n \rangle_c}{\sigma^{2n-2}}$$

are almost z-independent. In particular, if the density field is smoothed with a top-hat filter

Bernardeau' 94

$$S_3 = \frac{34}{7} + \gamma_1,$$

$$S_4 = \frac{60712}{1323} + \frac{62\,\gamma_1}{3} + \frac{7\,\gamma_1{}^2}{3} + \frac{2\,\gamma_2}{3},$$

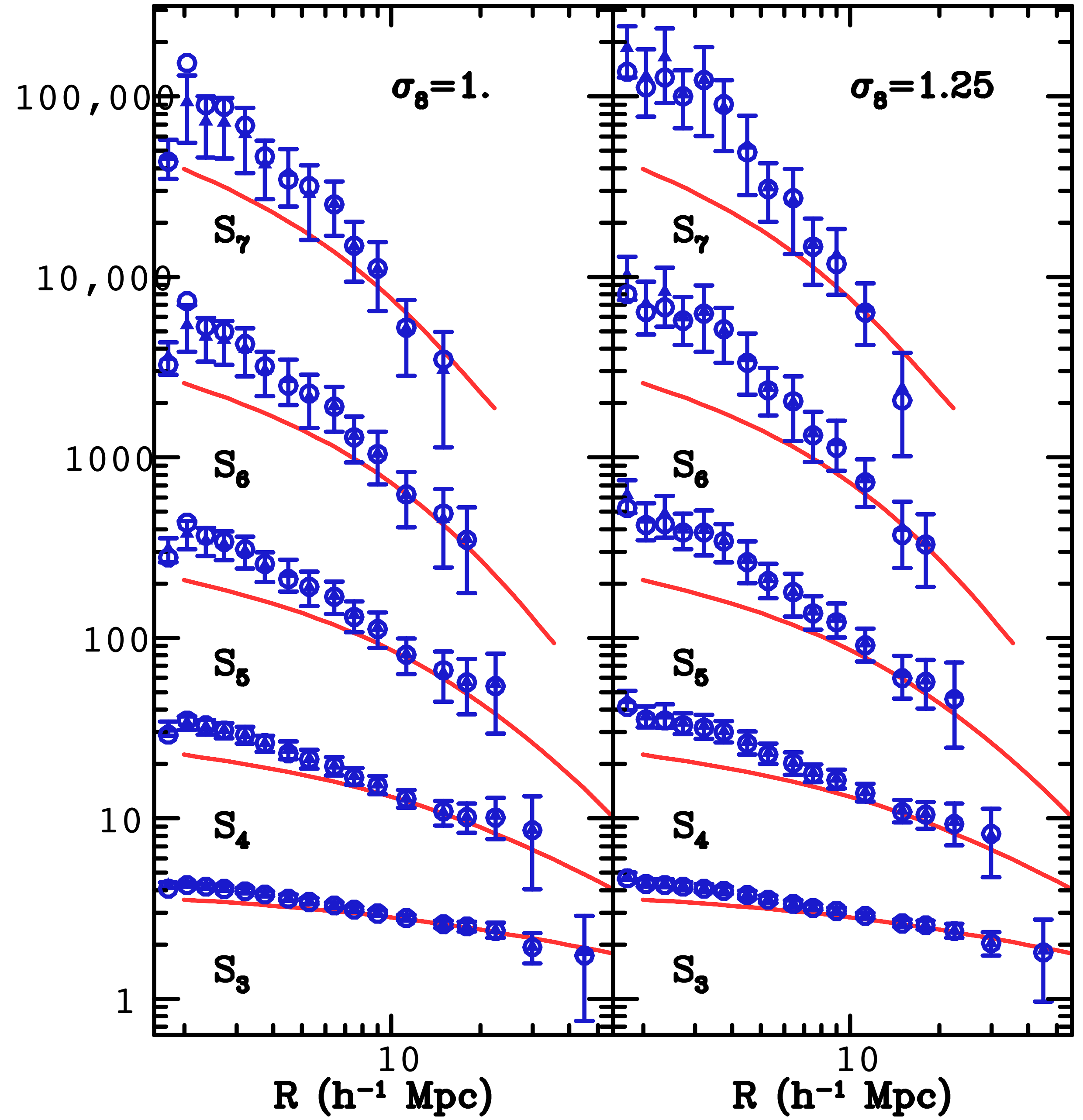
$$S_5 = \frac{200575880}{305613} + \frac{1847200\,\gamma_1}{3969} + \frac{6940\,\gamma_1{}^2}{63} + \frac{235\,\gamma_1{}^3}{27} + \frac{1490\,\gamma_2}{63} + \frac{50\,\gamma_1\,\gamma_2}{9} + \frac{10\,\gamma_3}{27},$$

where

$$\gamma_p = \frac{\mathrm{d}^p \log \sigma^2(R_0)}{\mathrm{d} \log^p R_0}$$

depends on the shape of the linear power spectrum.

Baugh & Gaztañaga '95



$$S_n = \frac{\langle \delta^n \rangle_c}{\sigma^{2n-2}}$$

From cumulants to PDF

The PDF of $x=\delta/\sigma$ can then be written as an Edgeworth expansion (in powers of σ):

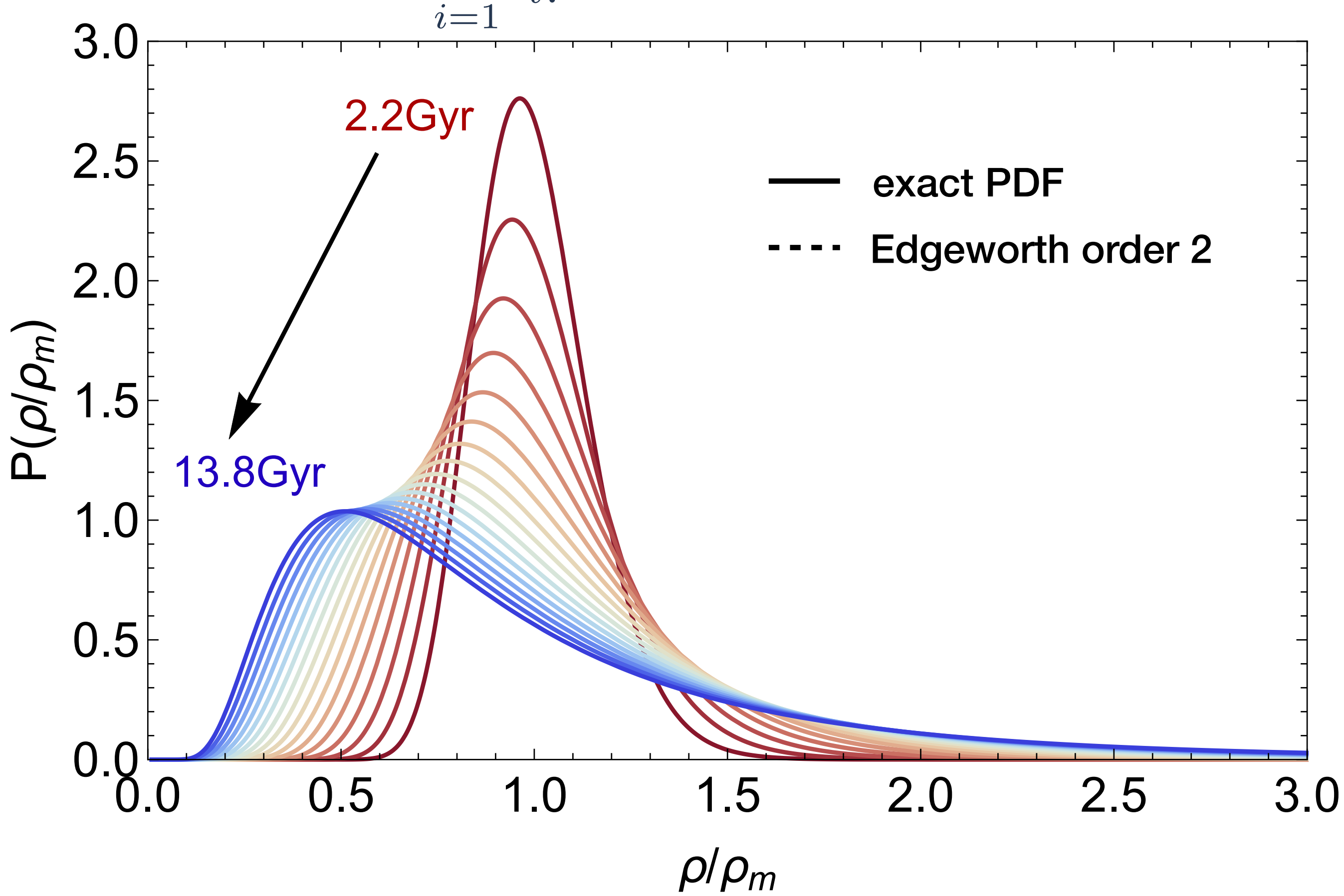
$$P(x) = G(x)\left[1 + \sigma \frac{S_3}{3!} H_3(x) + \sigma^2 \left(\frac{S_4}{4!} H_4(x) + \frac{1}{2}\left(\frac{S_3}{3!}\right)^2 H_6(x)\right) + \cdots\right]$$

which can be derived from the cumulant generating function of $\rho=1+\delta$

$$\exp \varphi(\lambda) = \int P(\rho) \exp(\lambda\rho) \leftrightarrow P(\rho) = \int_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\lambda}{2\imath\pi} \exp(\lambda\rho - \varphi(\lambda))$$

Laplace transform **inverse Laplace transform**

where $\varphi(\lambda) = \sum_{i=1}^{\infty} \frac{\lambda^i}{i!} \langle \rho^i \rangle_c$.



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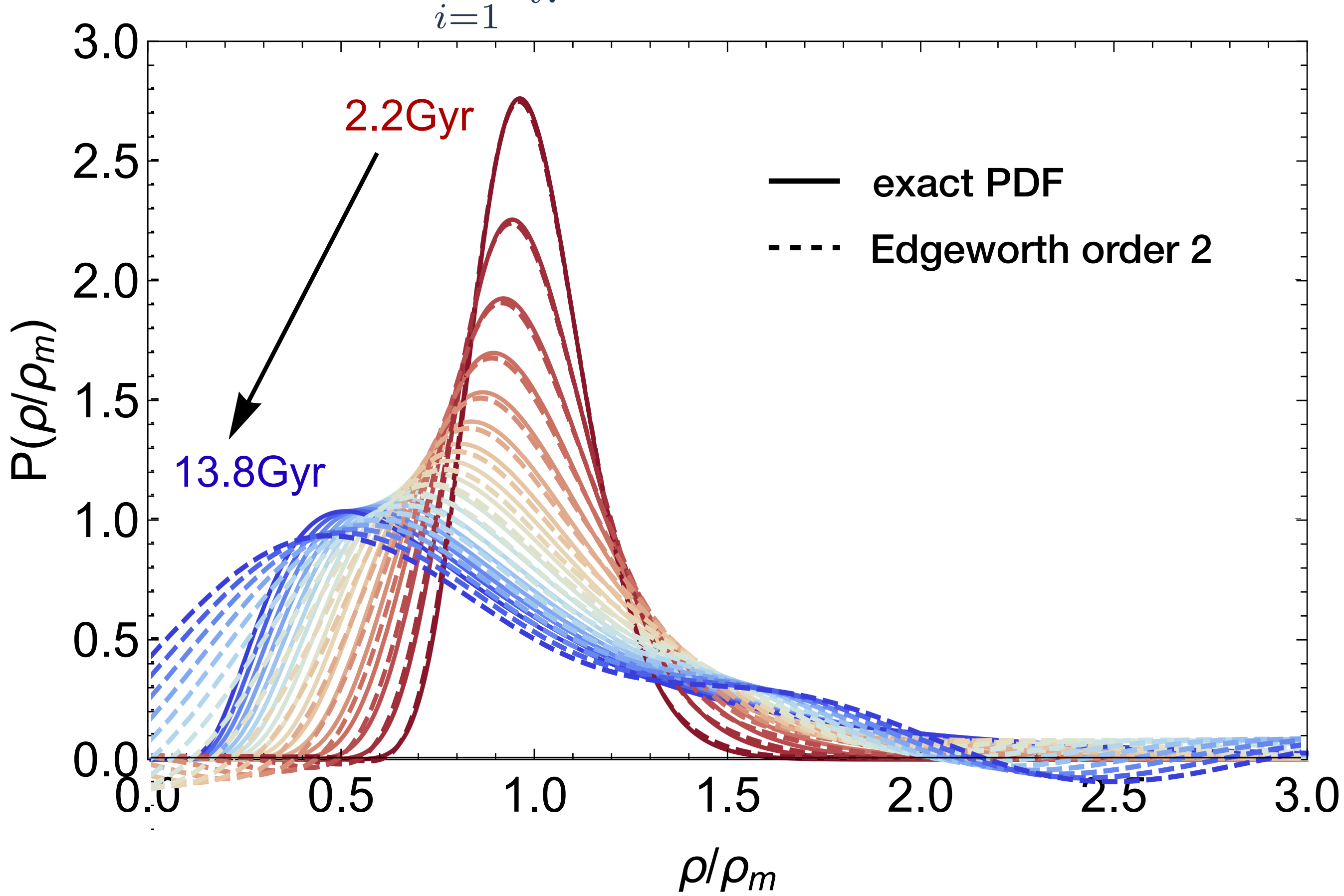
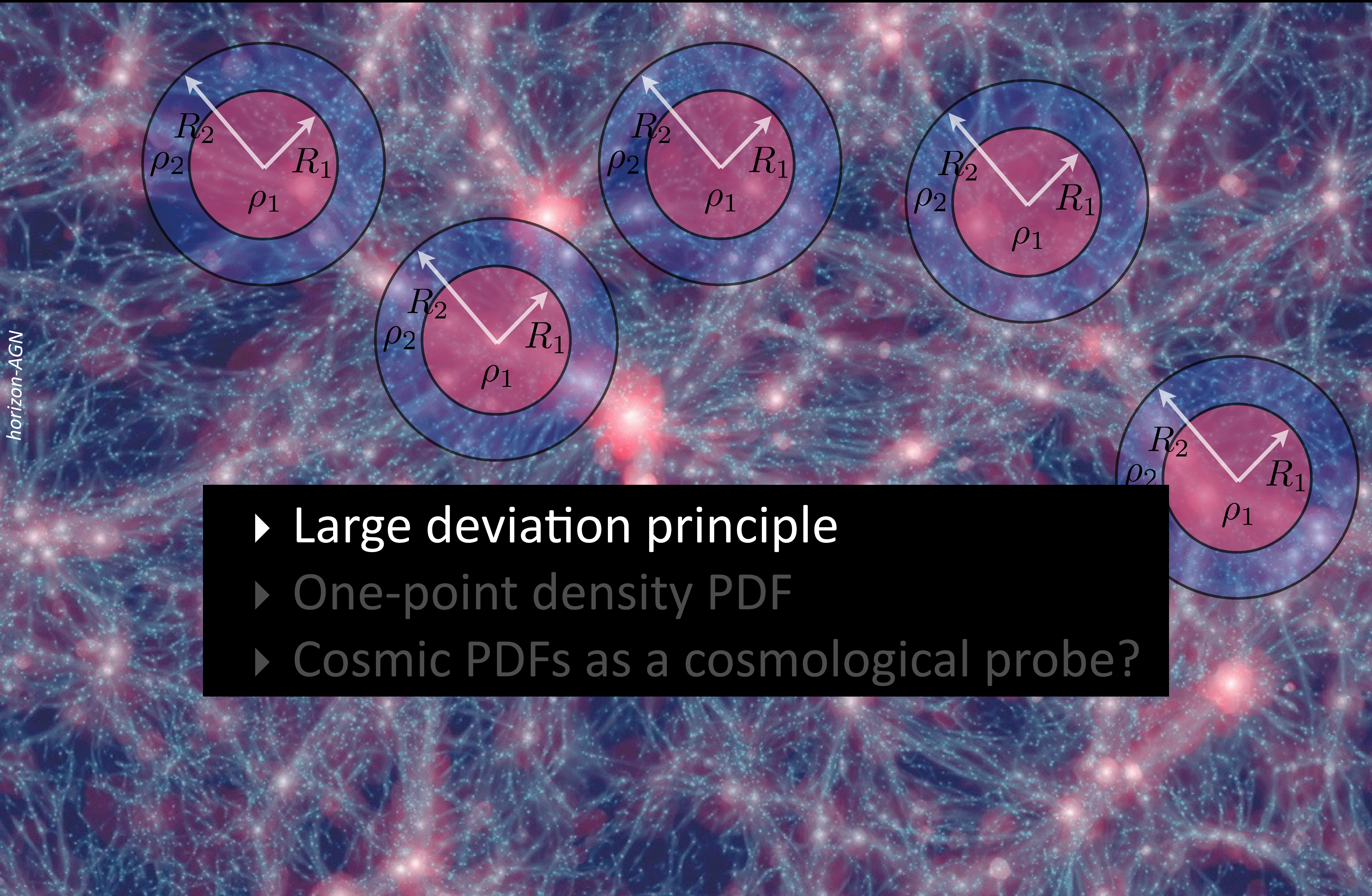


Problem : When this series is truncated at some orders, the PDF is unphysical : it is not normalised and can take negative values.

Solution : **large-deviation theory** provides us with a model for the PDF which does not suffer from those issues. All cumulants are exact at tree-order.

«An unlikely fluctuation is brought about by the least unlikely among all unlikely paths »

Beyond the power spectrum with large deviations theory

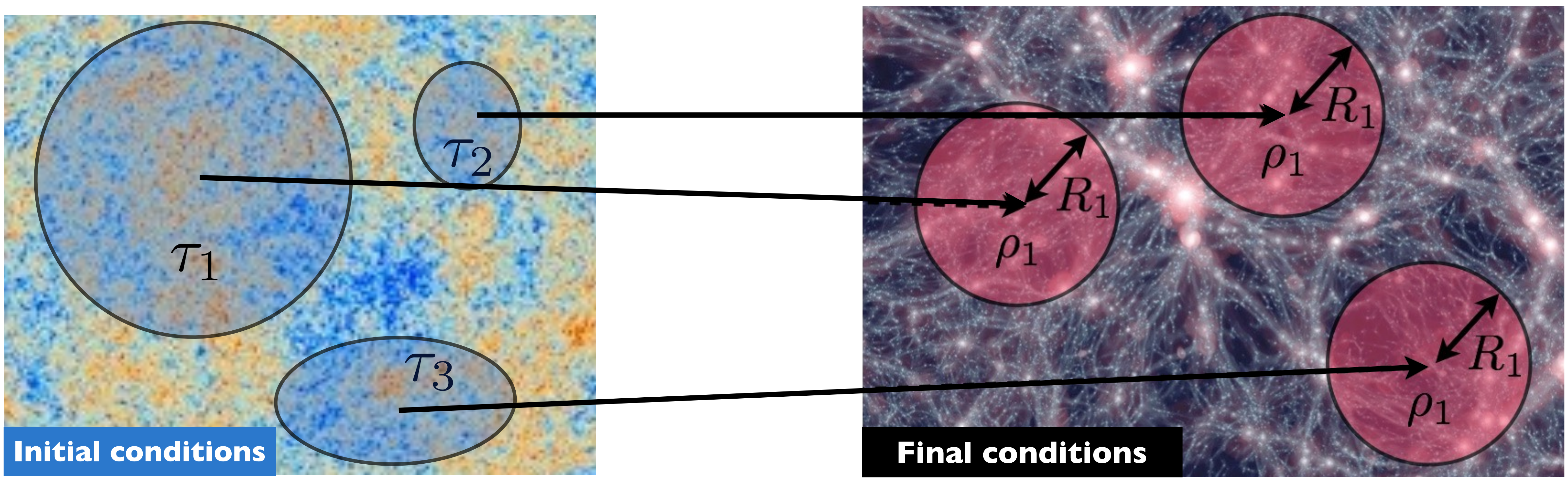


- Large deviation principle
- One-point density PDF
- Cosmic PDFs as a cosmological probe?

Bernardeau 94
Valageas 02
Bernardeau&Reimberg 16

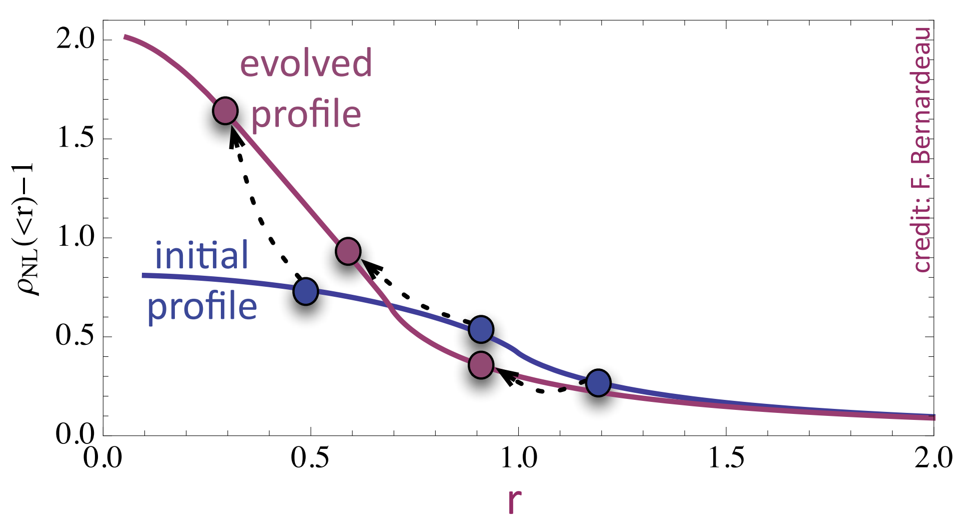
Large-deviation Theory: what is the most likely initial configuration a final density originates from?

In principle, one has to sum over all possible paths:



Different initial configurations can lead to the same final state! What is the most likely one?

Conjecture: Spherical symmetry enforces this most likely path to be the ***Spherical Collapse dynamics***.



$$\tau \longrightarrow \rho = \zeta_{\rm SC}(\tau)$$

$$r_0 \longrightarrow r = r_0 \rho^{-1/3}$$

Large-deviation Theory: in a nutshell

LDP tells us how to compute the **cumulant generating function** from the initial conditions using the spherical collapse as the « mean dynamics »:

$$\varphi(\{\lambda_k\}) = \sup_{\rho_i}(\lambda_i \rho_i - I(\rho_i))$$

Varadhan's theorem

Why?

$$\varphi(\lambda_k) = \Big\langle \underbrace{\exp(\Sigma_i \lambda_i \rho_i)}_{\simeq \lambda_i \langle \rho_i \rangle + \lambda_i \lambda_j \langle \rho_i \rho_j \rangle + \dots} \Big\rangle = \int_0^\infty \Pi_i \mathrm{d}\rho_i P(\{\rho_k\}) \exp\left(\Sigma_i \lambda_i \rho_i\right)$$

initial density contrast

$$= \int \mathcal{D}\left[\tau(\vec{x})\right] \mathcal{P}\left[\tau(\vec{x})\right] \exp(\lambda_i \rho_i \left[\tau(\vec{x})\right])$$

known Gaussian PDF $\mathcal{P}(\tau) \propto e^{-I(\tau)}$

contraction principle

$$= \int \mathrm{d}\tau_i \exp\left(\lambda_i \zeta_{\mathrm{SC}}(\tau_i) - I(\tau_i)\right)$$

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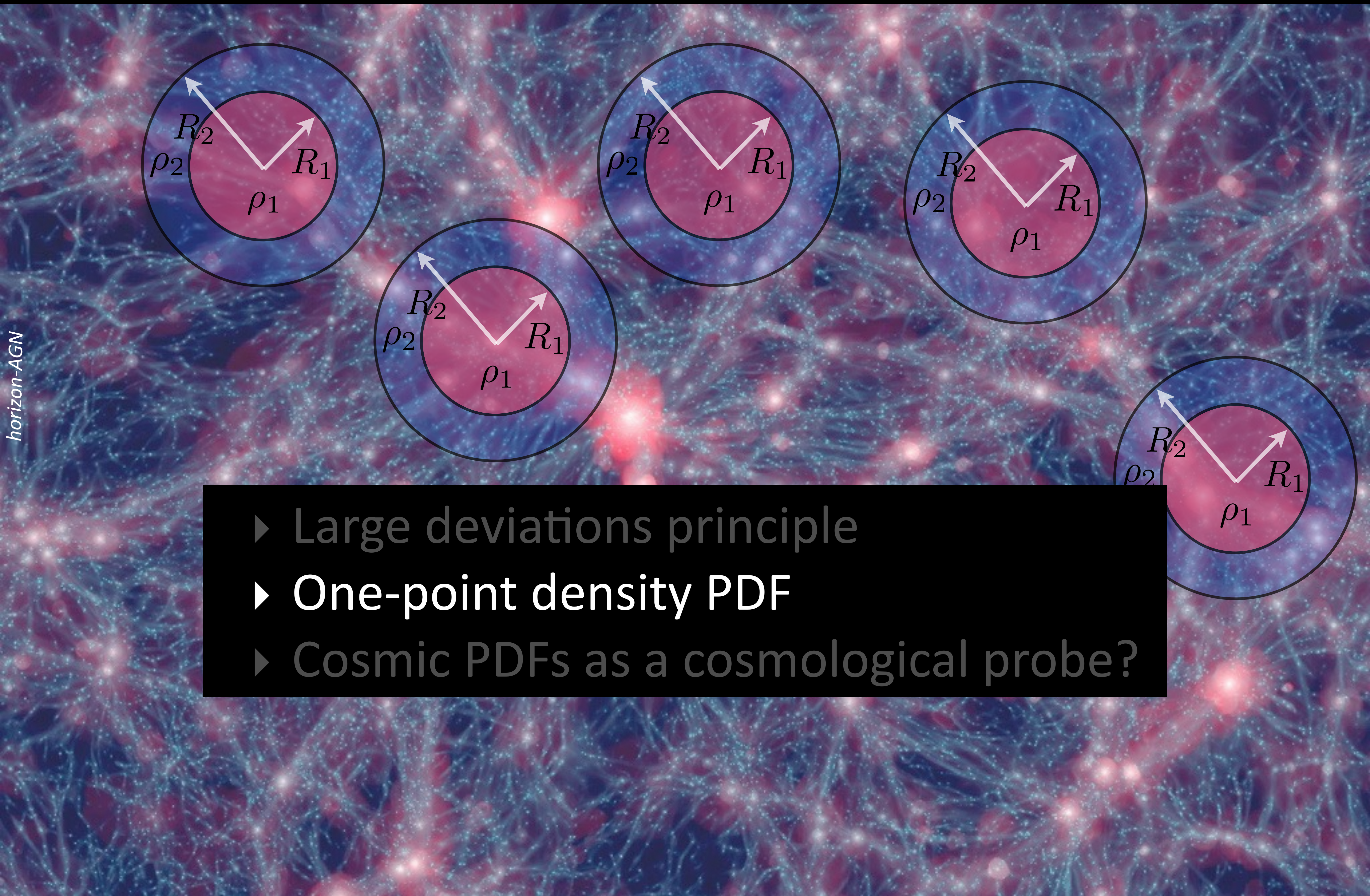
Varadhan's theorem

The density **PDF** is then obtained via an inverse Laplace transform of the CGF

$$\exp\,\varphi(\lambda) = \int P(\rho)\exp(\lambda\rho) \leftrightarrow P(\rho) = \int_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\lambda}{2\imath\pi}\exp(\lambda\rho - \varphi(\lambda))$$

- This is exact in the zero variance limit. We then extrapolate to non zero values.
- **Parameter-free** theory which depends on cosmology through : the spherical collapse dynamics, the linear power spectrum and growth of structure.
- Predictions are fully **analytical** if one applies the LDP to the log. *(Uhlemann+16)*

Beyond the power spectrum with large deviations theory



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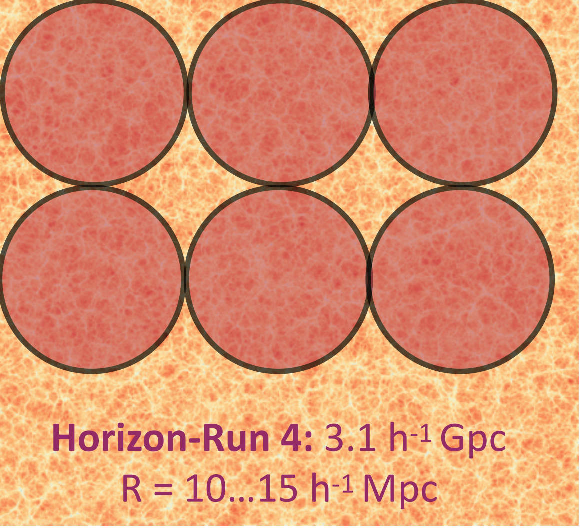
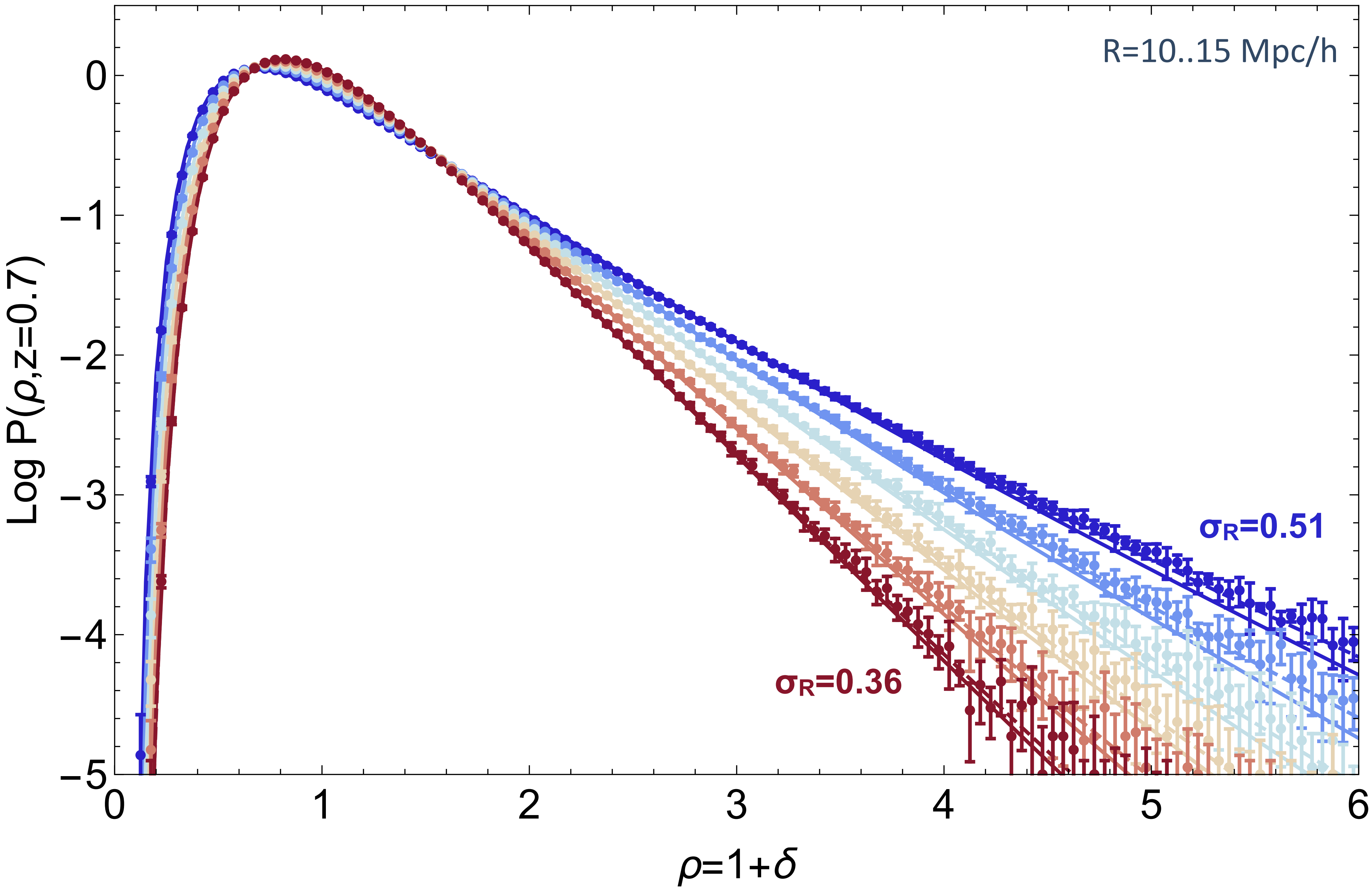
- One-point density PDF
- Cosmic PDFs as a cosmological probe?



One-cell density PDF



SC+16b

We have developed a fast and easy-to-use public code…

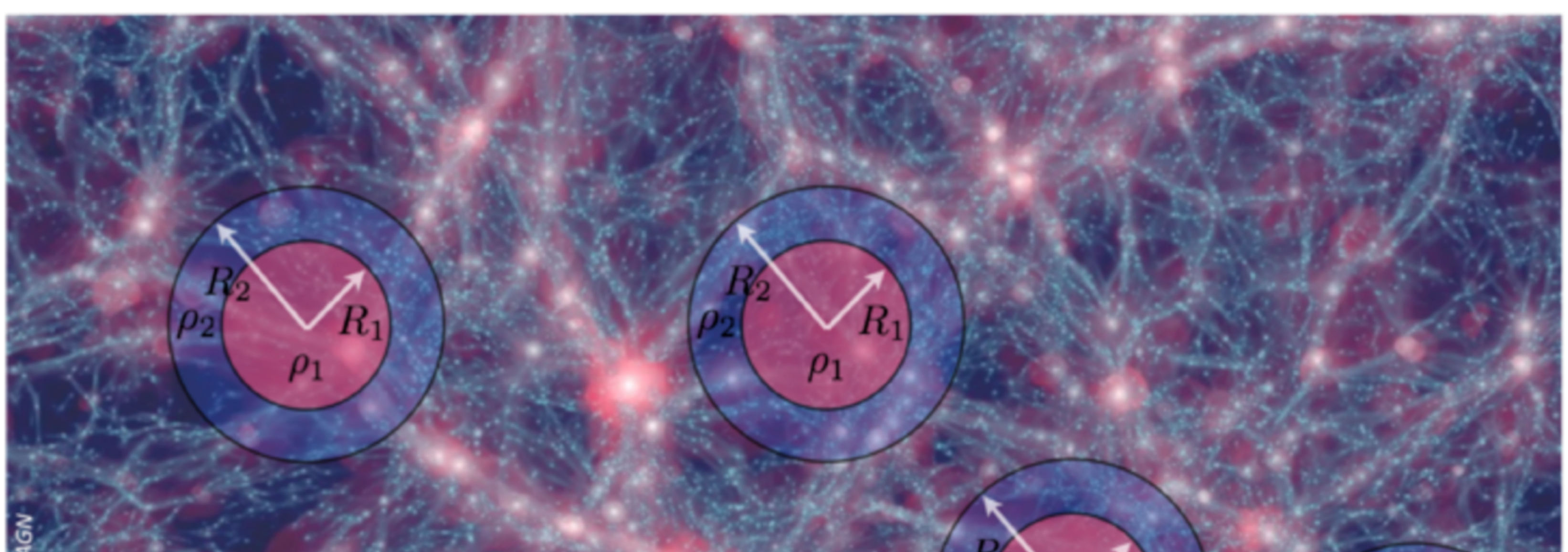
http://cita.utoronto.ca/~codis/LSSFast.html

LSSFAST

A Mathematica package to compute cosmic density PDF in the large-deviation regime

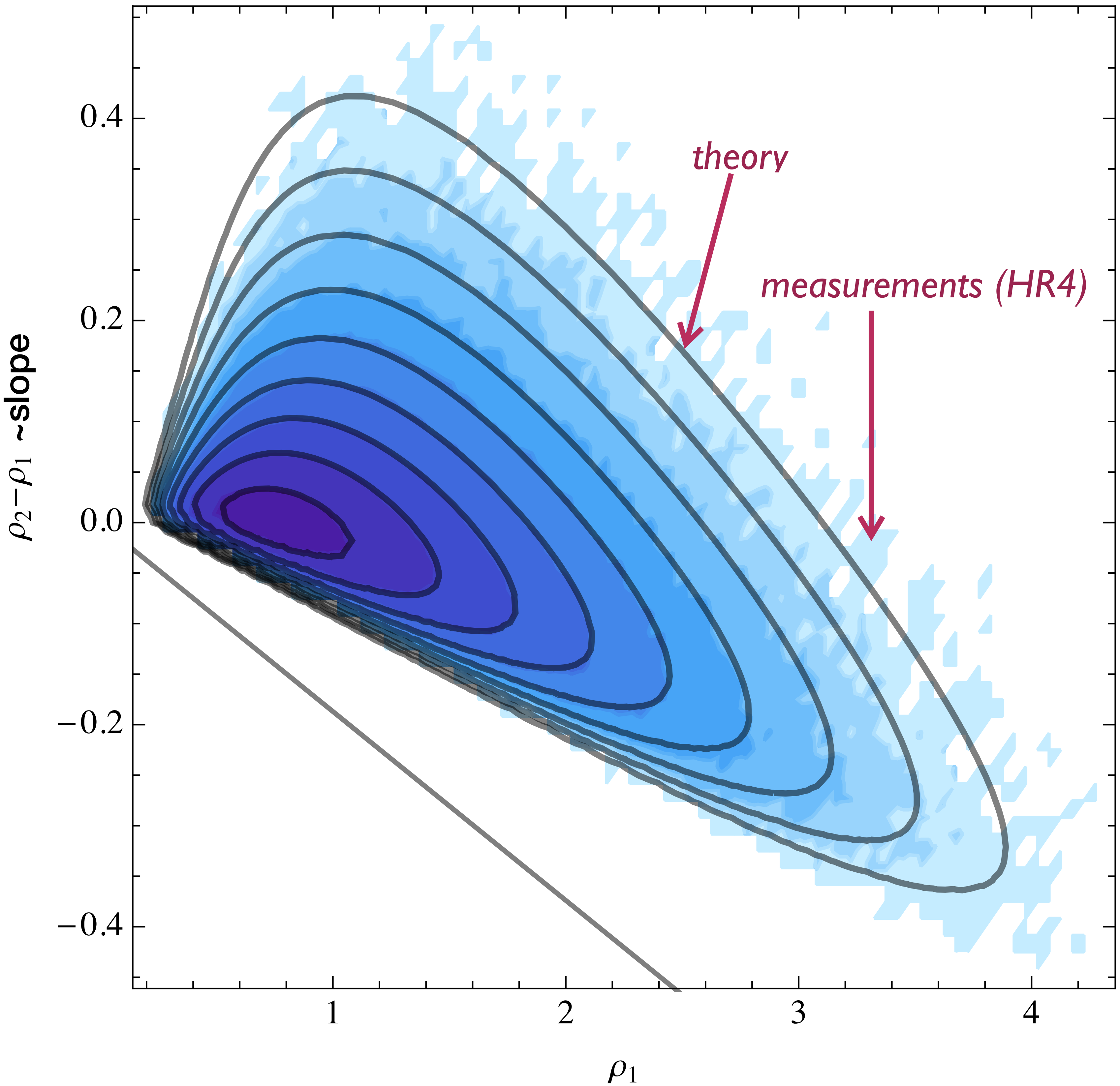
Author: Sandrine Codis (CITA)
Last modified: 04/03/2016
Code: LSSFast.tar.gz
This code is based on theoretical works in collaboration with Francis Bernardeau (IAP, CEA-Saclay) , Christophe Pichon (IAP, KIAS) and Cora Uhlemann (Utrecht University)

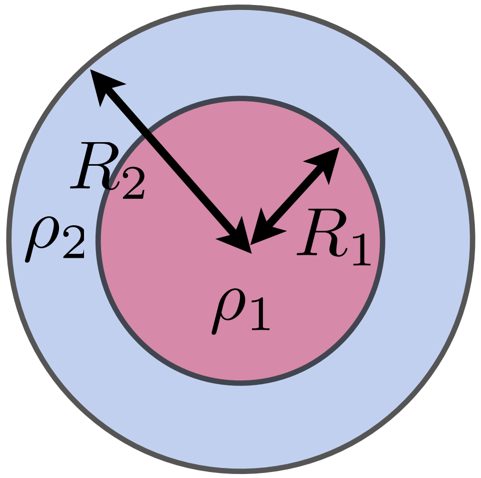
The LSSFast code is a free software distributed under the terms of the GNU-General Public License 3. It can be redistributed and modified at your own risk.
This program is made publicly available in the hope that it will be useful in scientific research but without any warranty.
The companion paper "Constraining the nature of dark energy via density PDF" by S. Codis, F. Bernardeau, C. Pichon, C. Uhlemann and S. Prunet illustrates the possible use of LSSFast for cosmological data analysis.
Any questions or remarks can be emailed to codis@cita.utoronto.ca



Bernardeau+15
Uhlemann+16

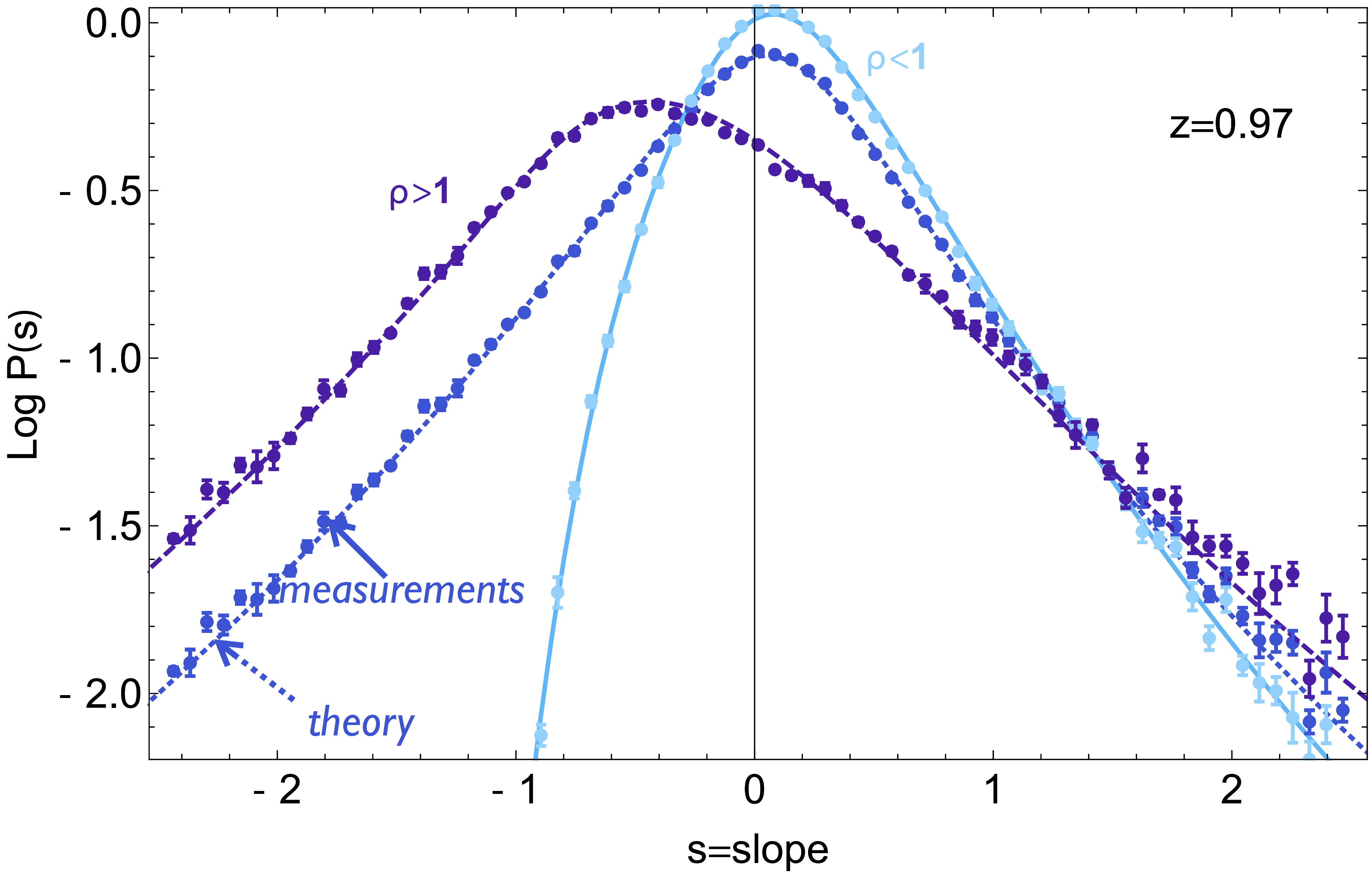
Two-cell PDF

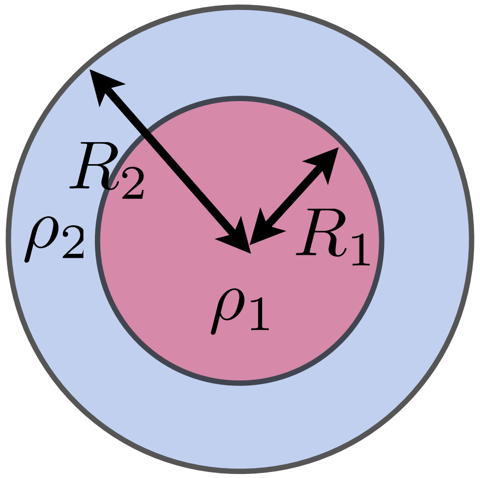




Bernardeau+15
Uhlemann+16

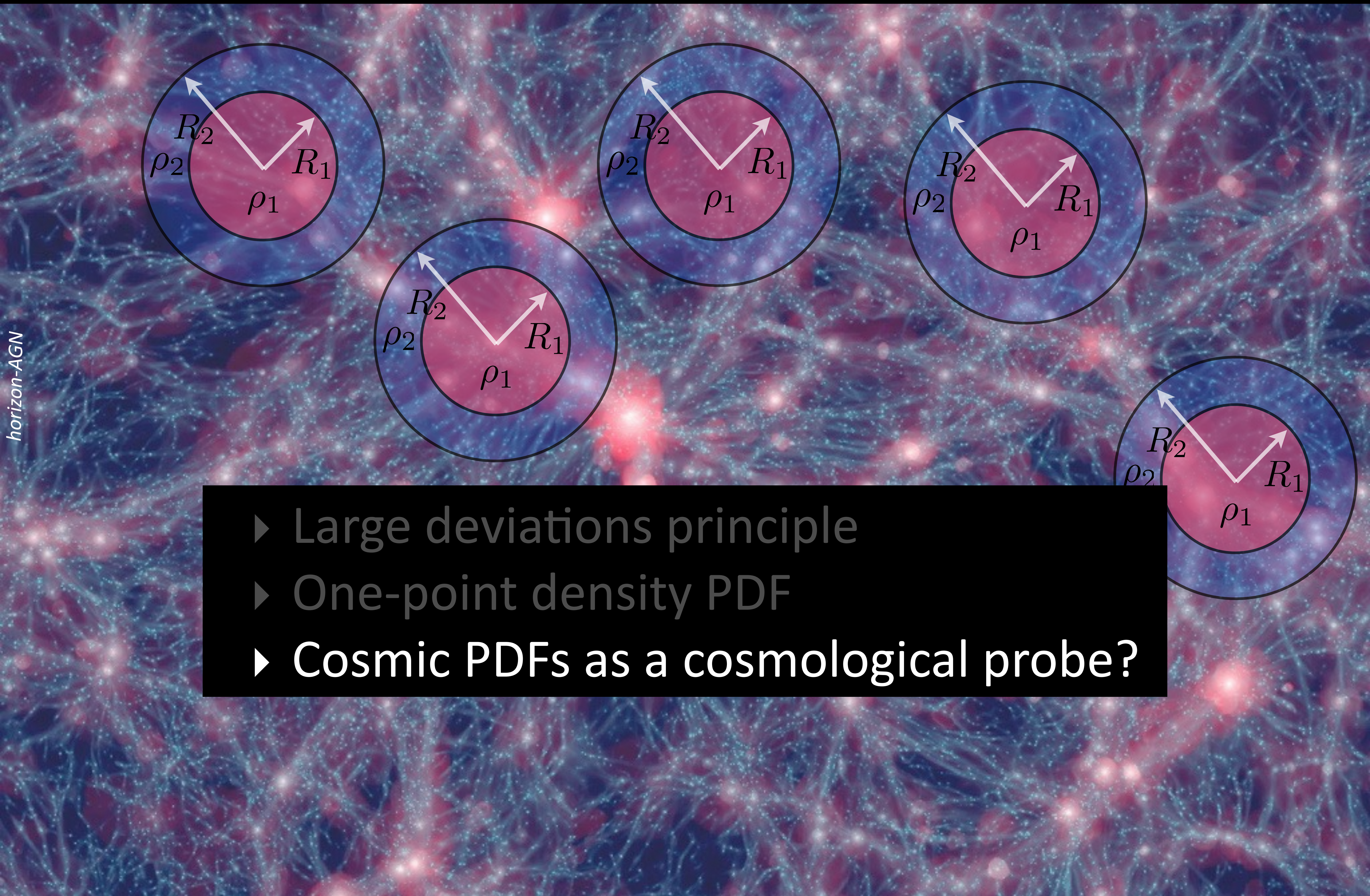
Two-cell PDF
statistics of the slope





Higher density environments have more negative slopes (peaks!).

Beyond the power spectrum with large deviations theory



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Where is the cosmology dependence?

To get one-cell PDF, one has to:

1) know the rate function of the initial conditions e.g (Gaussian):

$$I(\tau(R_0)) = \sigma^2(R_p) \times 1/2\tau(R_0)^2/\sigma^2(R_0)$$

where the initial variance is a function of the **linear power spectrum**

$$\sigma^2(R) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k}\, P_{\mathrm{lin}}(k) W_{\mathrm{TH}}^2(kR)$$

2) deduce the rate function of the final densities from the Contraction Principle

$$I(\rho) = I(\tau = \zeta^{-1}(\rho))$$

spherical collapse dynamics

3) compute CGF and then PDF

$$\boxed{P(\rho|\nu, P_{\mathrm{lin}}, \sigma_{\mathrm{NL}}(R,z))}$$

growth of structure dark energy

modifications of gravity

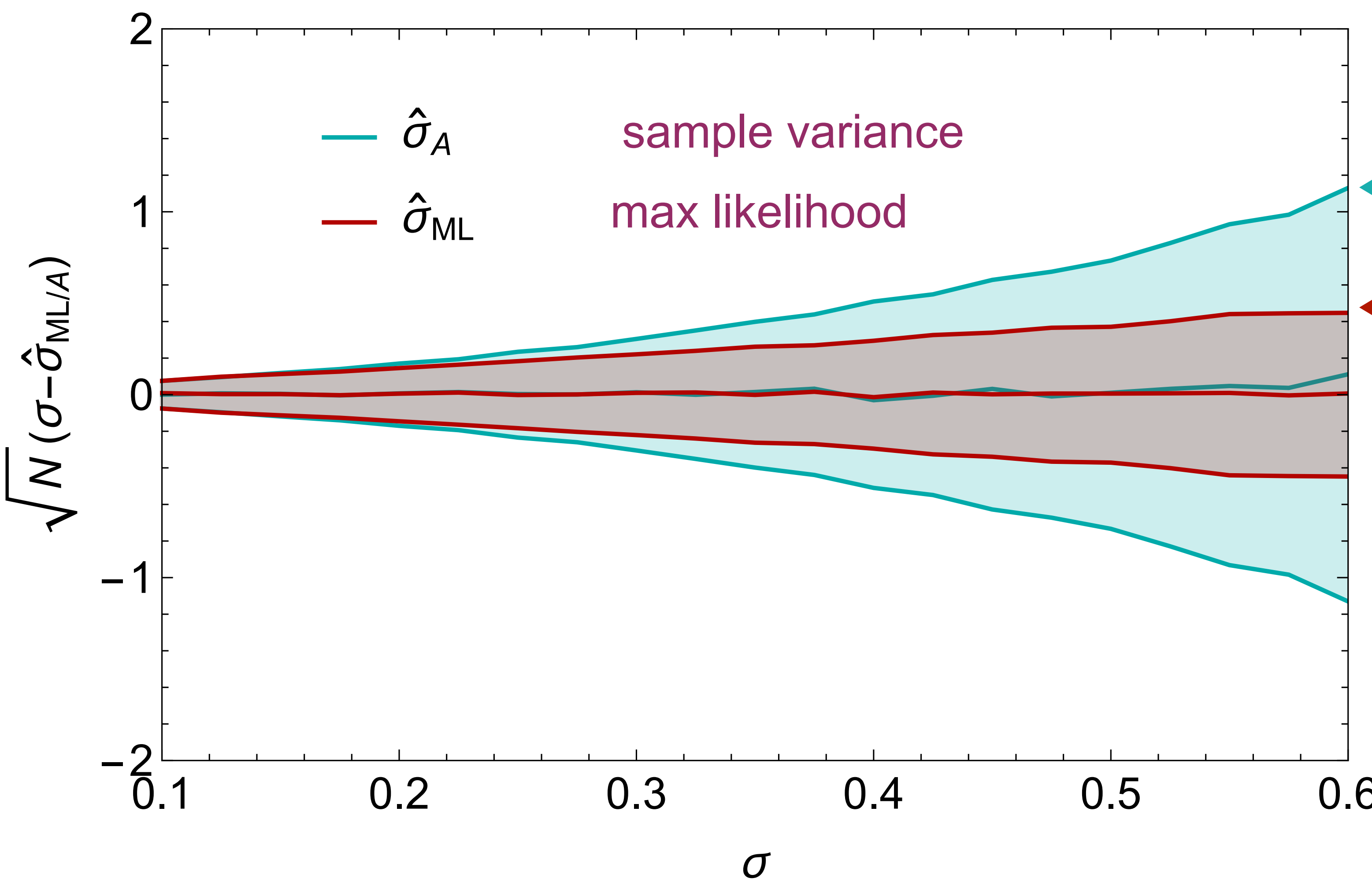
initial statistics primordial non-Gaussianities

ML estimator for the variance

The full knowledge of the PDF can be used to estimate the redshift evolution of the density variance σ and therefore the DE e.o.s through D(z).

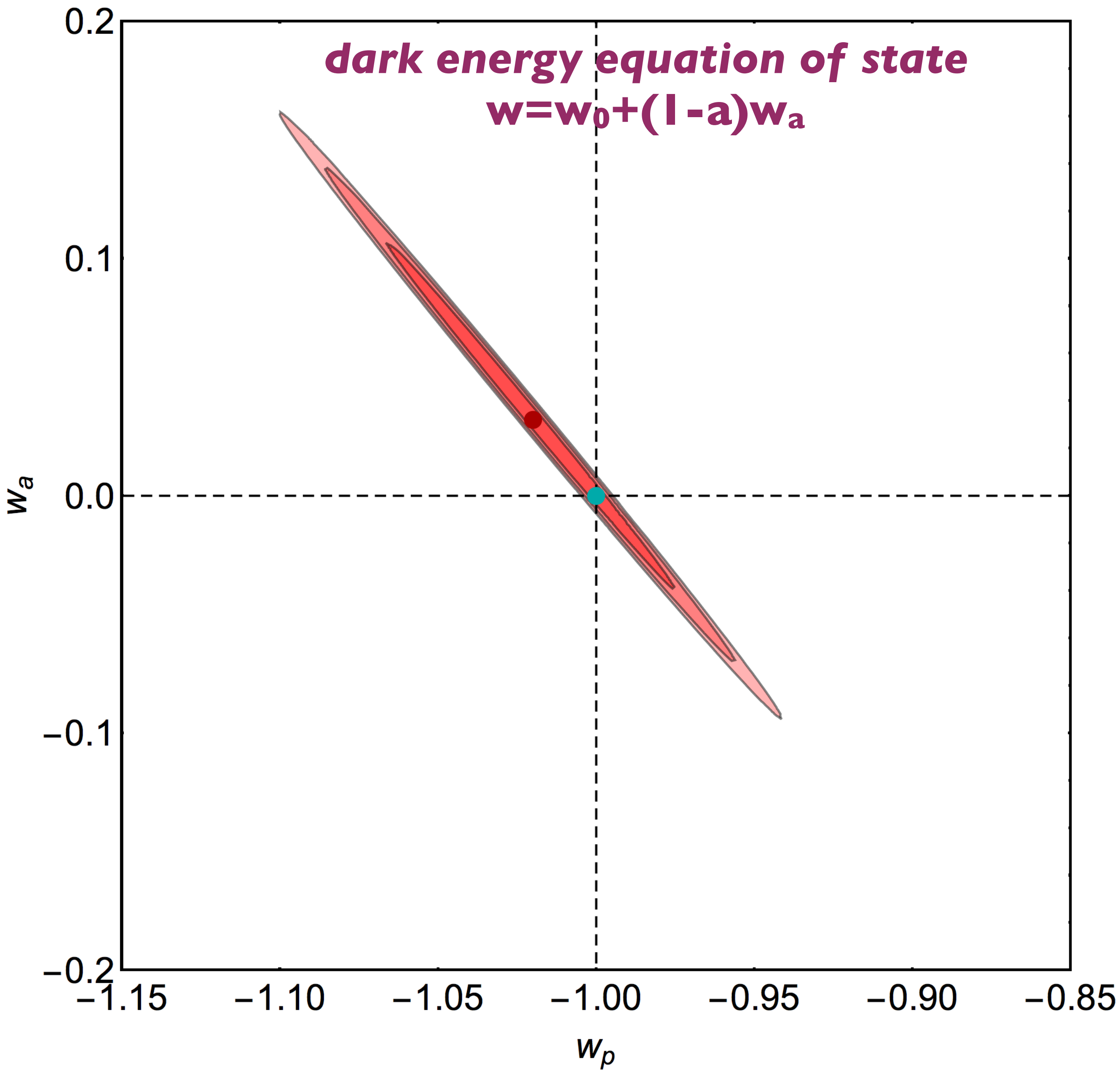
Maximum Likelihood estimator : $\hat{\sigma}^2_{\mathrm{ML}} = \mathrm{argmax}_{\tilde{\sigma}^2} \prod_{i=1}^{\mathrm{N}} \mathcal{P}(\rho_i|\tilde{\sigma}^2)$

Sample variance : $\hat{\sigma}^2_A = \frac{1}{N} \sum_{i=1}^{\mathrm{N}} (\rho_i - 1)^2$



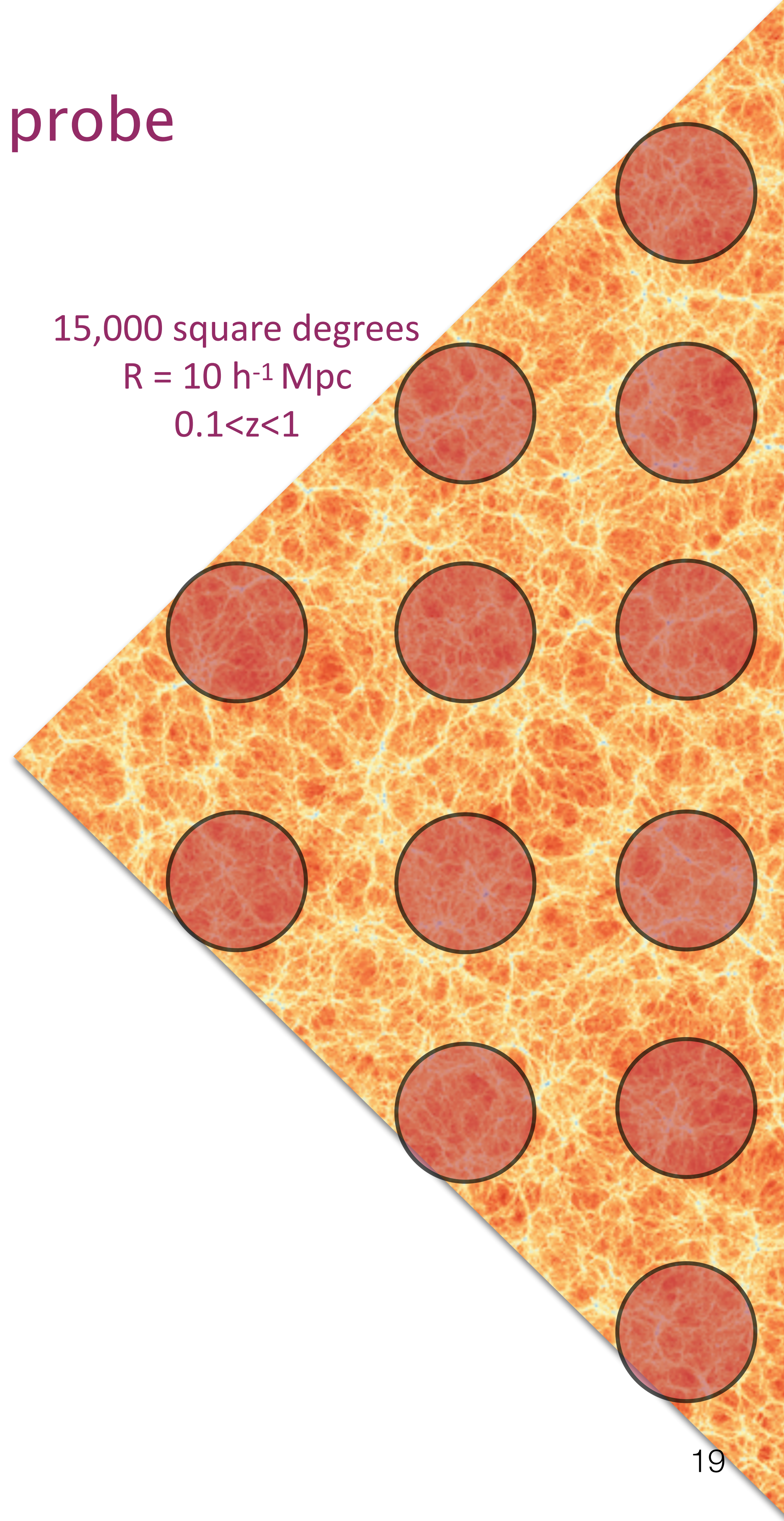
When the PDF becomes non-Gaussian (high σ), the sample variance is sub-optimal compared to the ML estimator

PDF as a cosmological probe



FoM for a Euclid-like survey

15,000 square degrees
R = 10 h^{-1} Mpc
0.1<z<1



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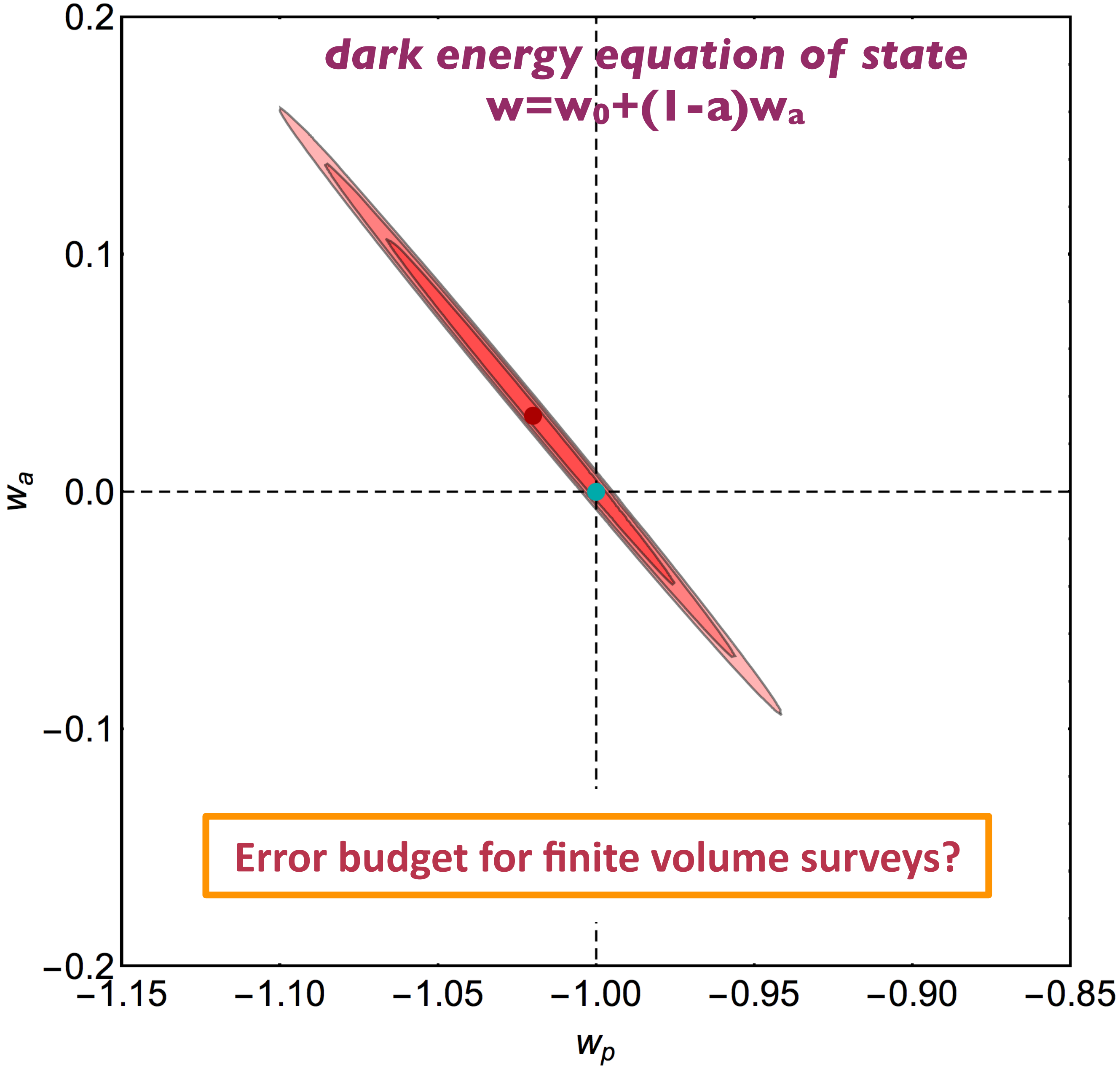

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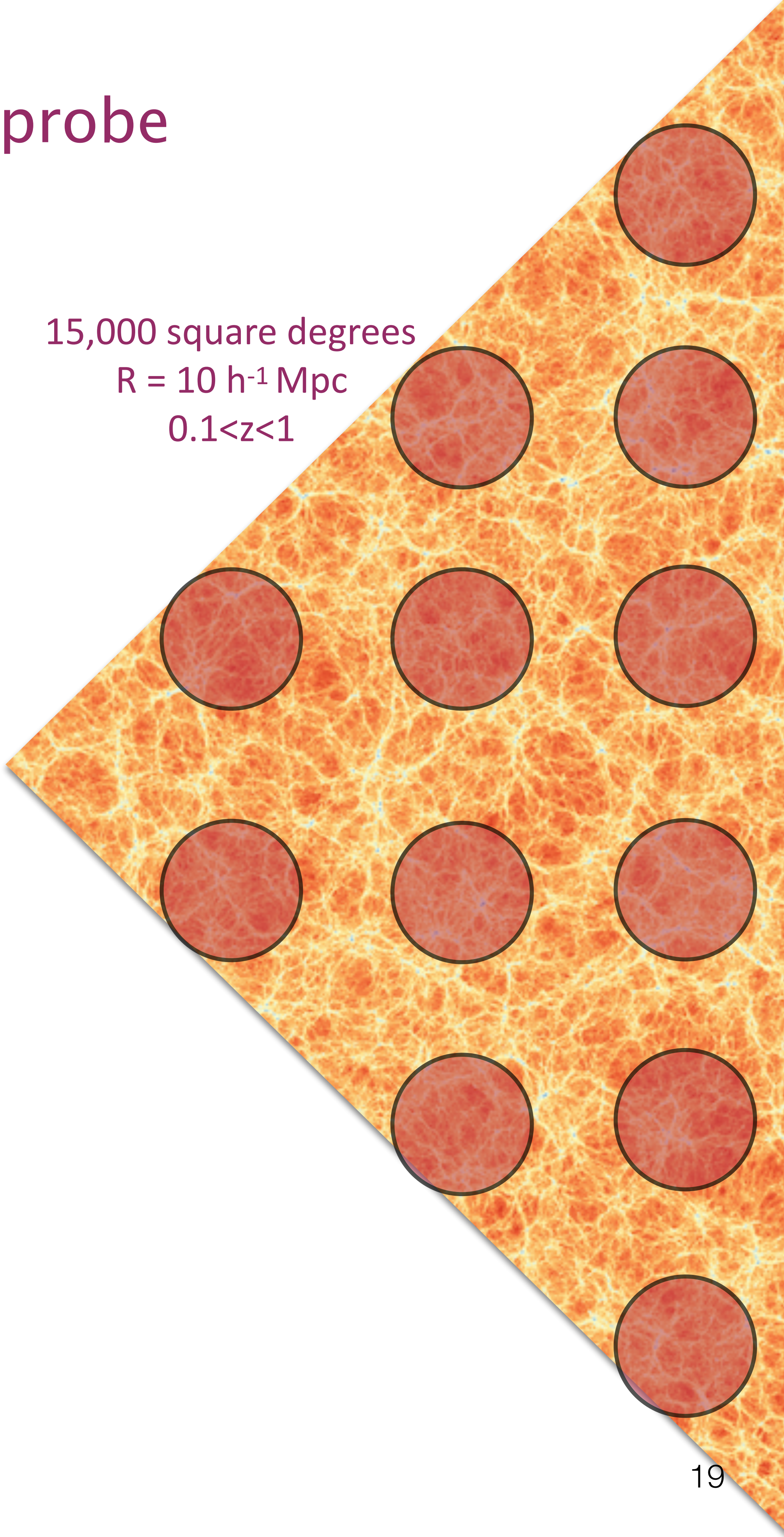
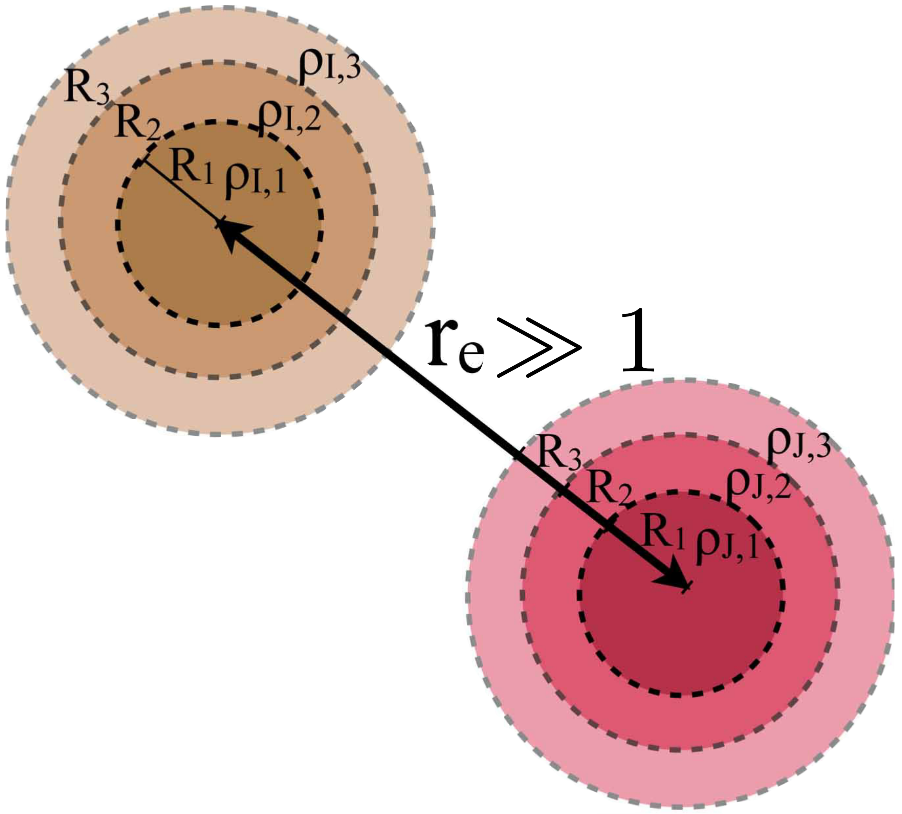


SC+16a

Error budget?



Maximum likelihood requires proper handling of **correlations** between spheres at **finite** separations.
The large-deviation principle provides a framework to compute the expected two-point correlations in the (not so) large separation limit

dark matter correlation **density bias**

$$P(\rho(x), \rho'(x + r_e)) = P(\rho)P(\rho')[1 + \xi(r_e)b(\rho)b(\rho')]$$

where the large-deviations bias is

spherical collapse

$$b(\rho) = \frac{\zeta_{\mathrm{SC}}^{-1}(\rho)}{\sigma^2(R\rho^{1/3})}$$

encodes $P_{\mathrm{lin}}(k)$

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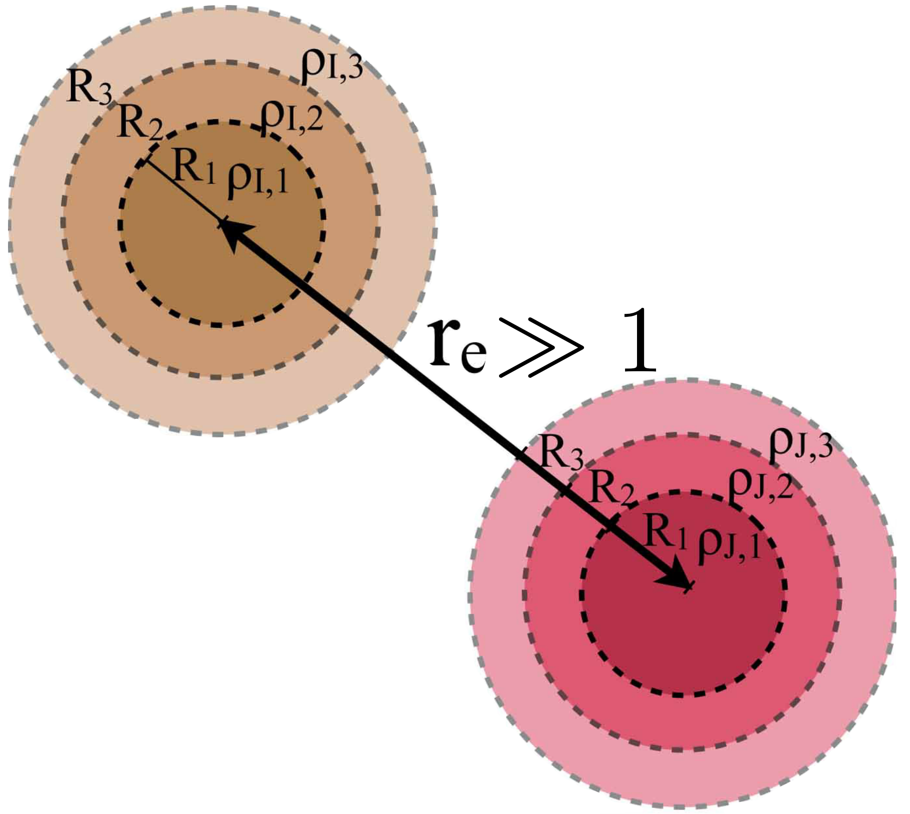

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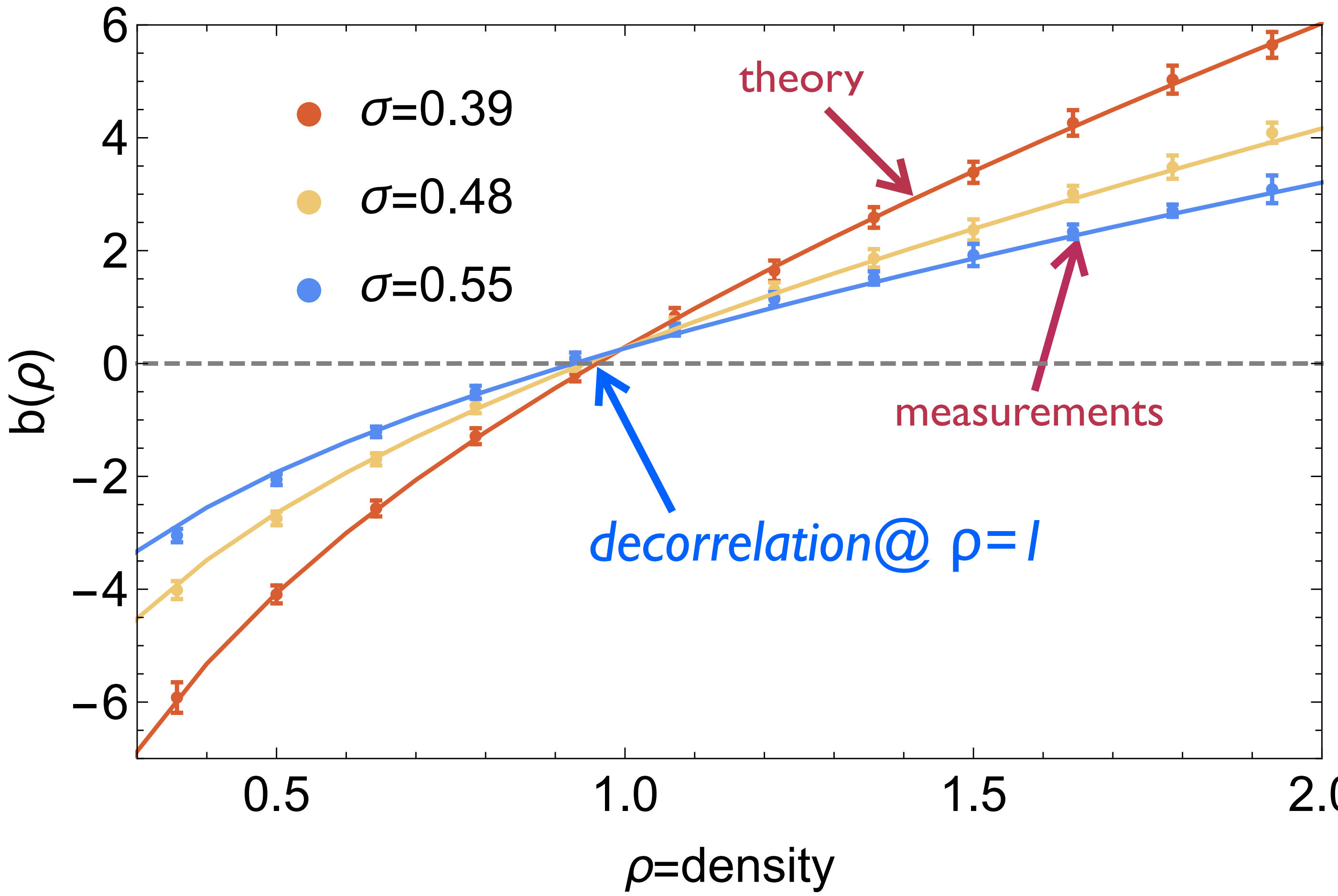

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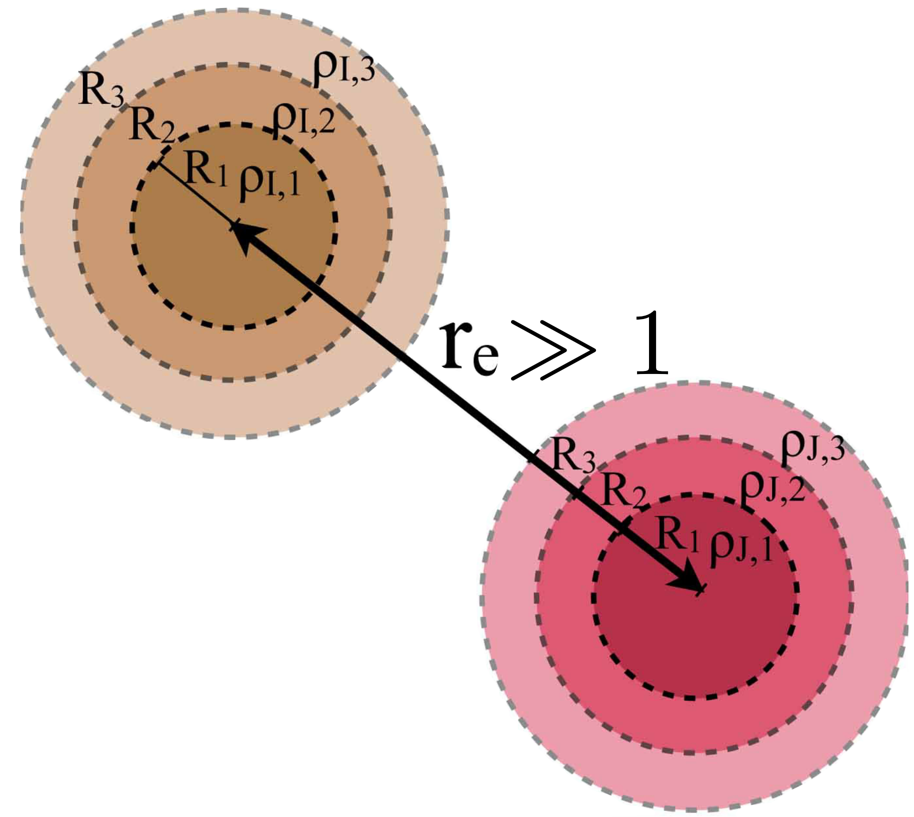

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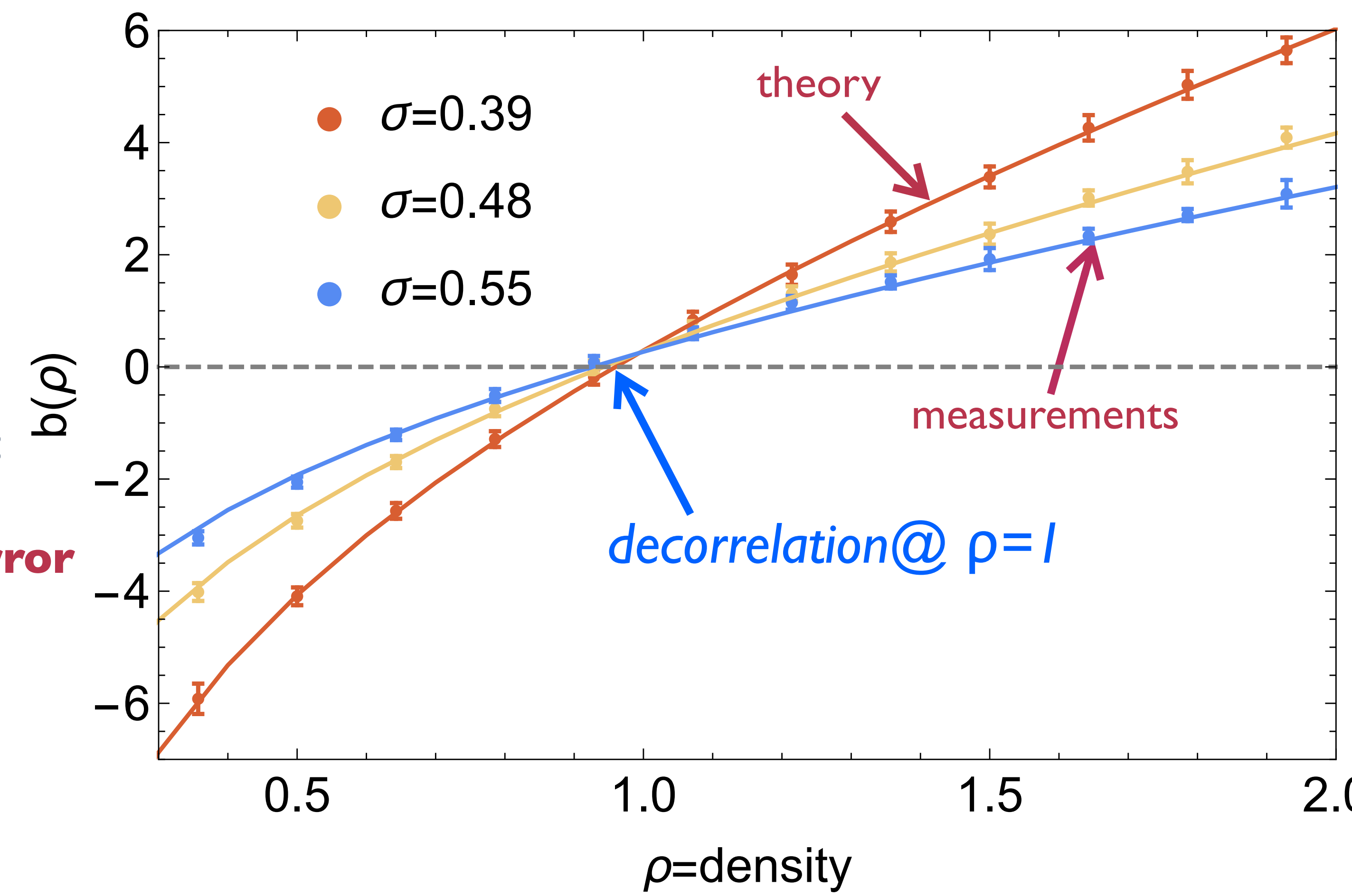
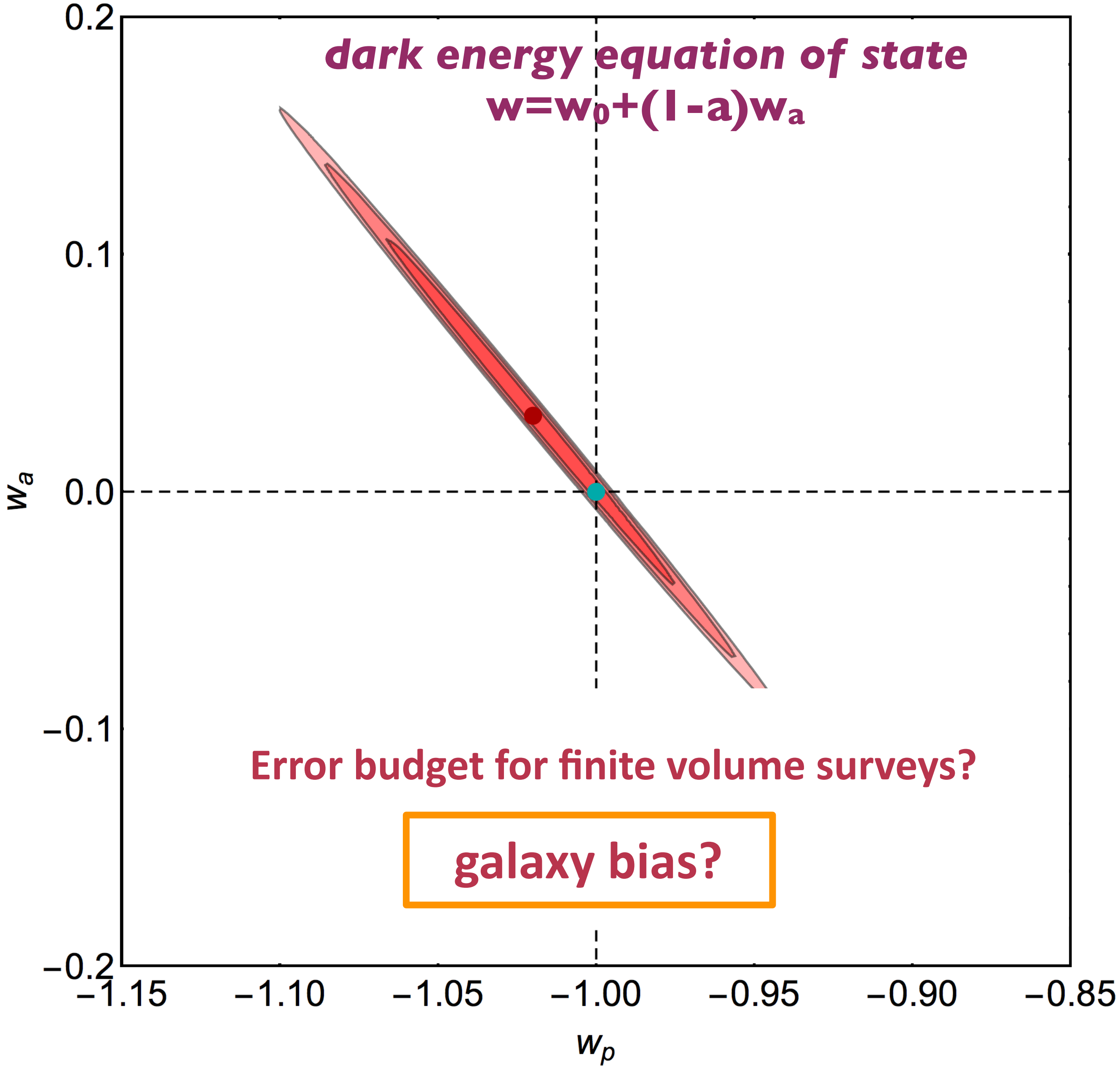


The typical cosmic variance on the density PDF is then:

shot noise **finite volume error**

$$\langle \hat{\mathcal{P}}^2(\rho)\rangle - \langle \hat{\mathcal{P}}(\rho)\rangle^2 = \frac{\mathcal{P}(\rho)}{N\Delta\rho} + \xi b^2(\rho)\mathcal{P}^2(\rho)$$

PDF as a cosmological probe



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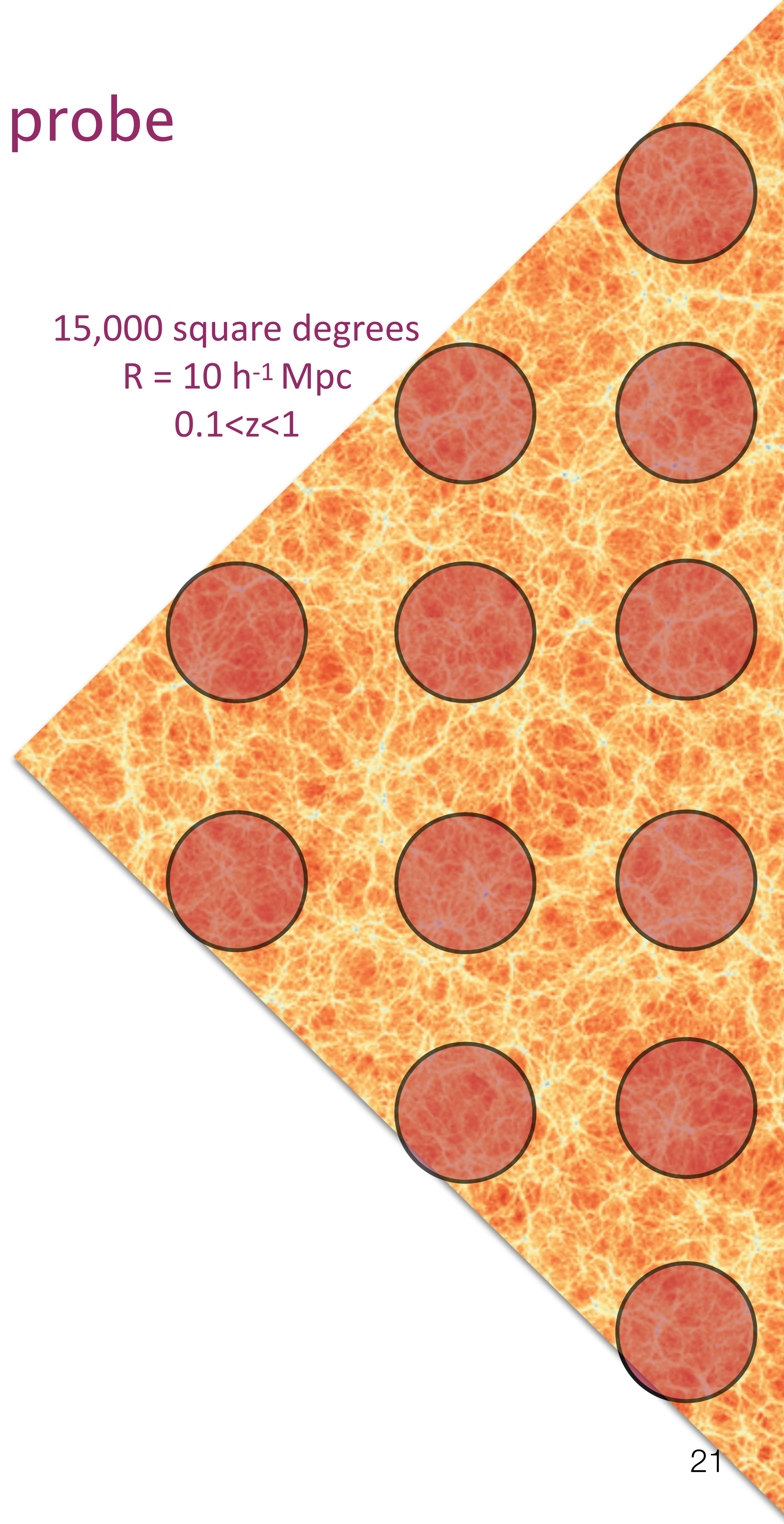
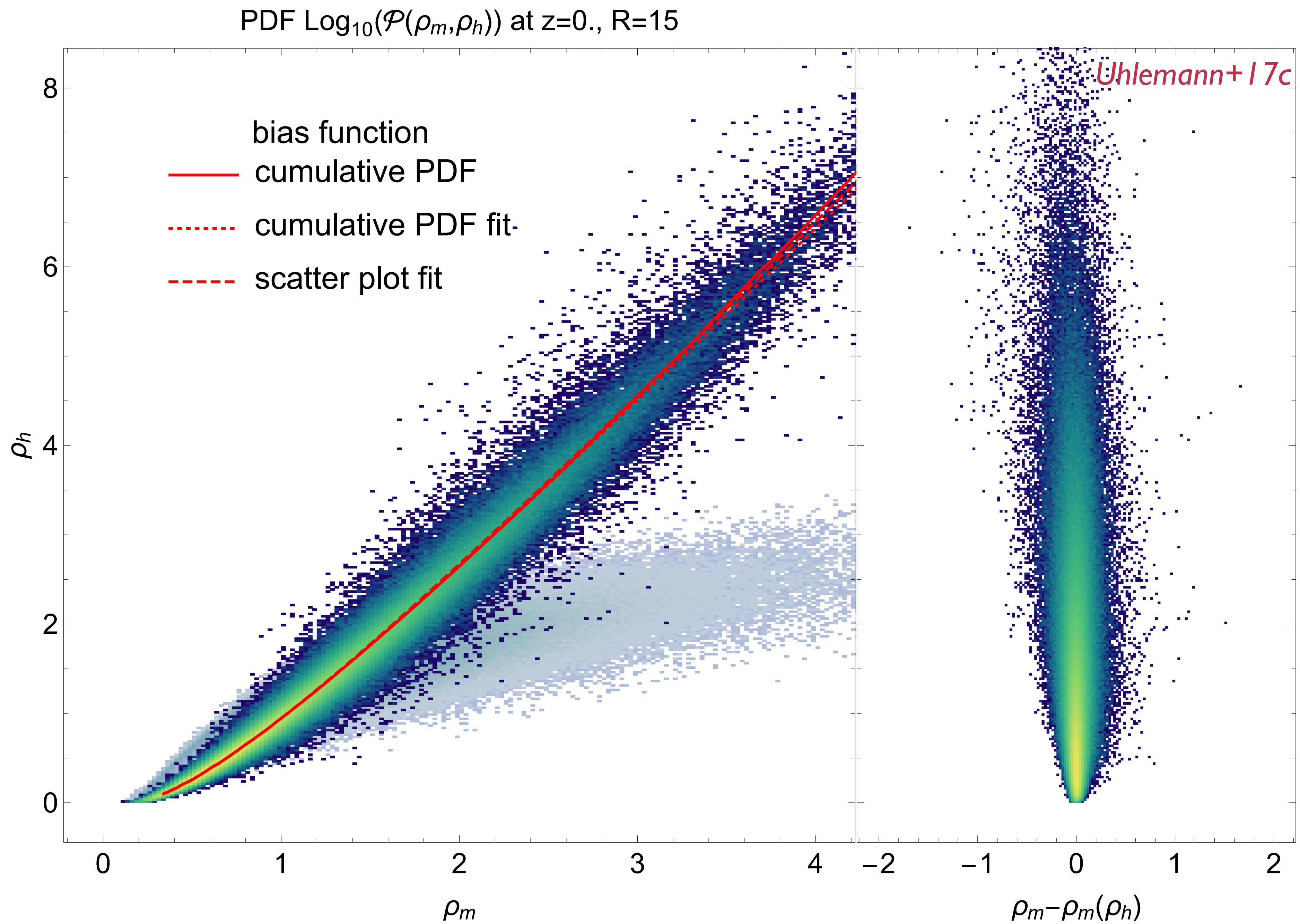


How to deal with biased tracers?

Halo bias can be accounted for and marginalised over for cosmological experiments…

We use a quadratic log bias model: $\log \rho_m = b_0 + \beta_1 \sigma \log \rho_h + \beta_2 \sigma \log^2 \rho_h$



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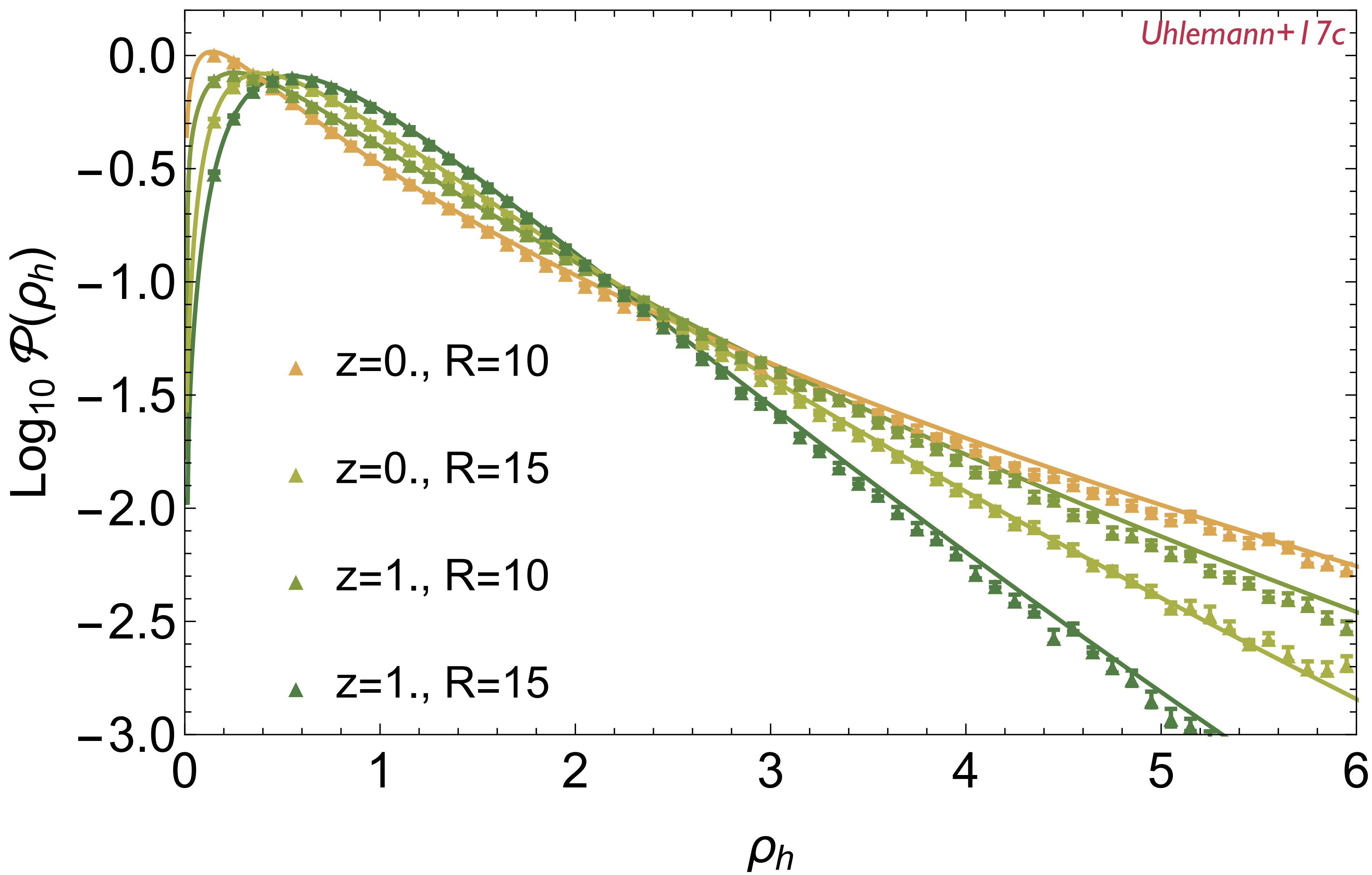

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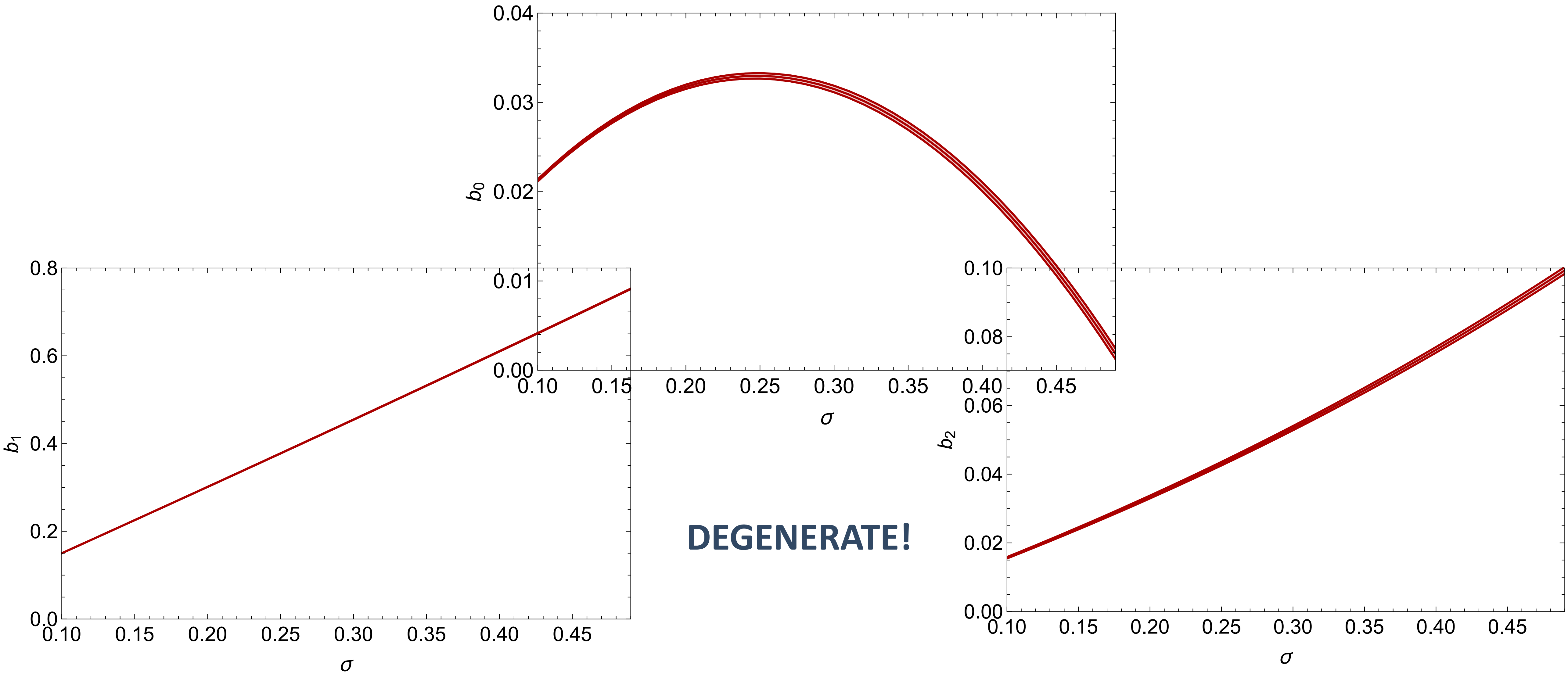

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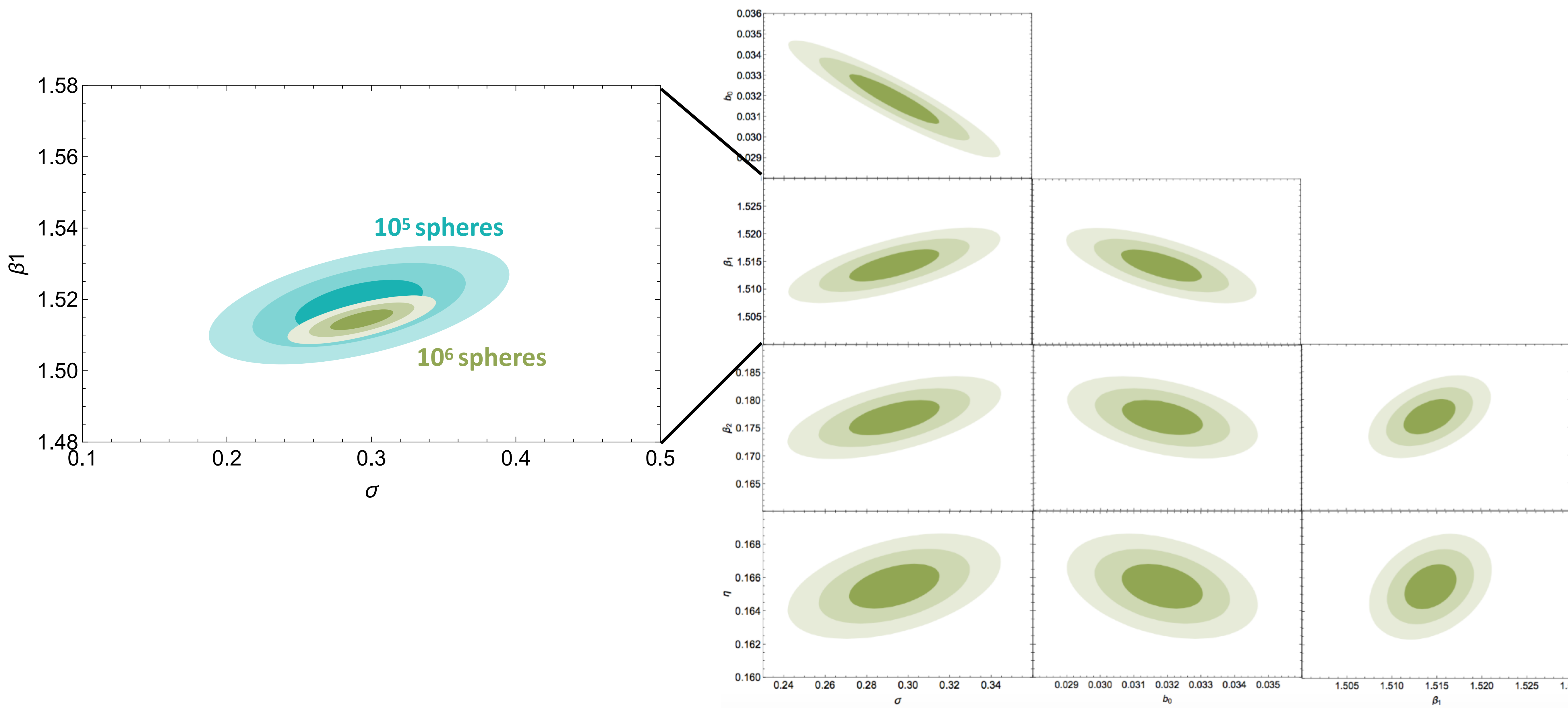
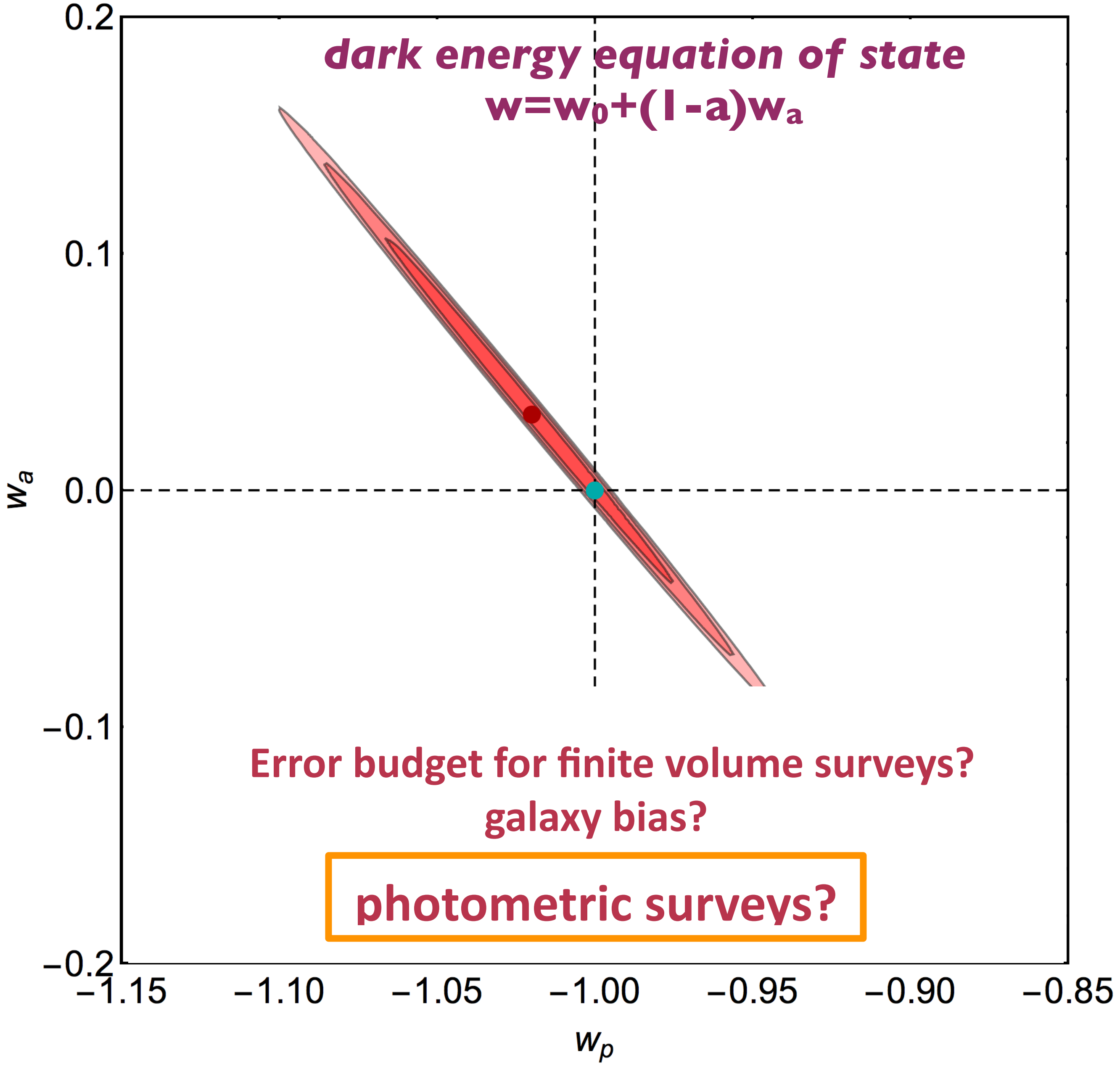
+ 2pt PDF



PDF as a cosmological probe



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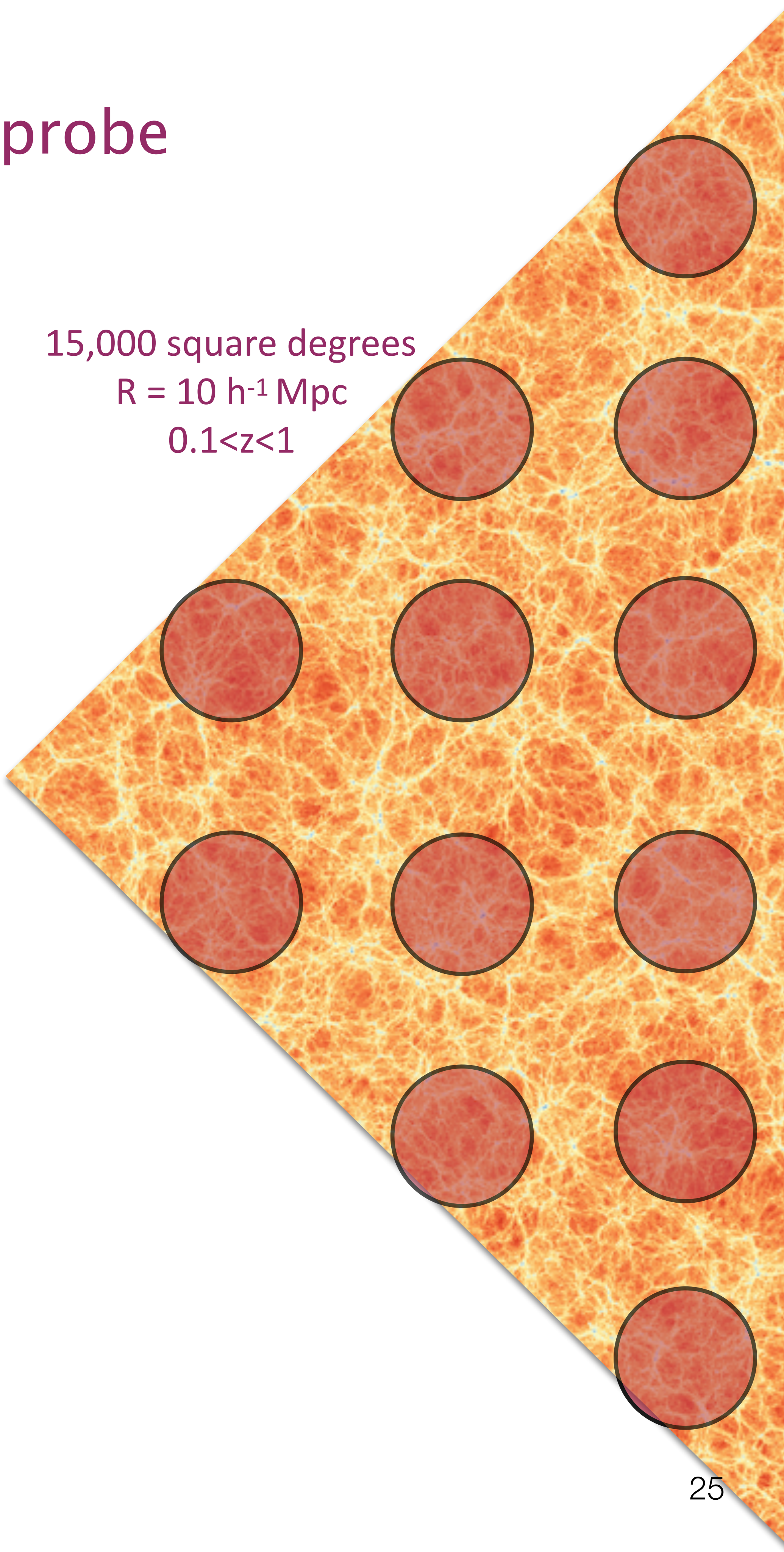
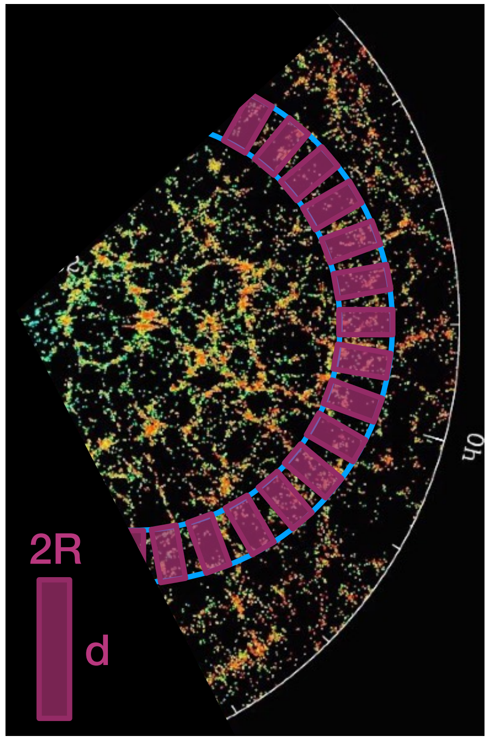


Uhlemann+17e

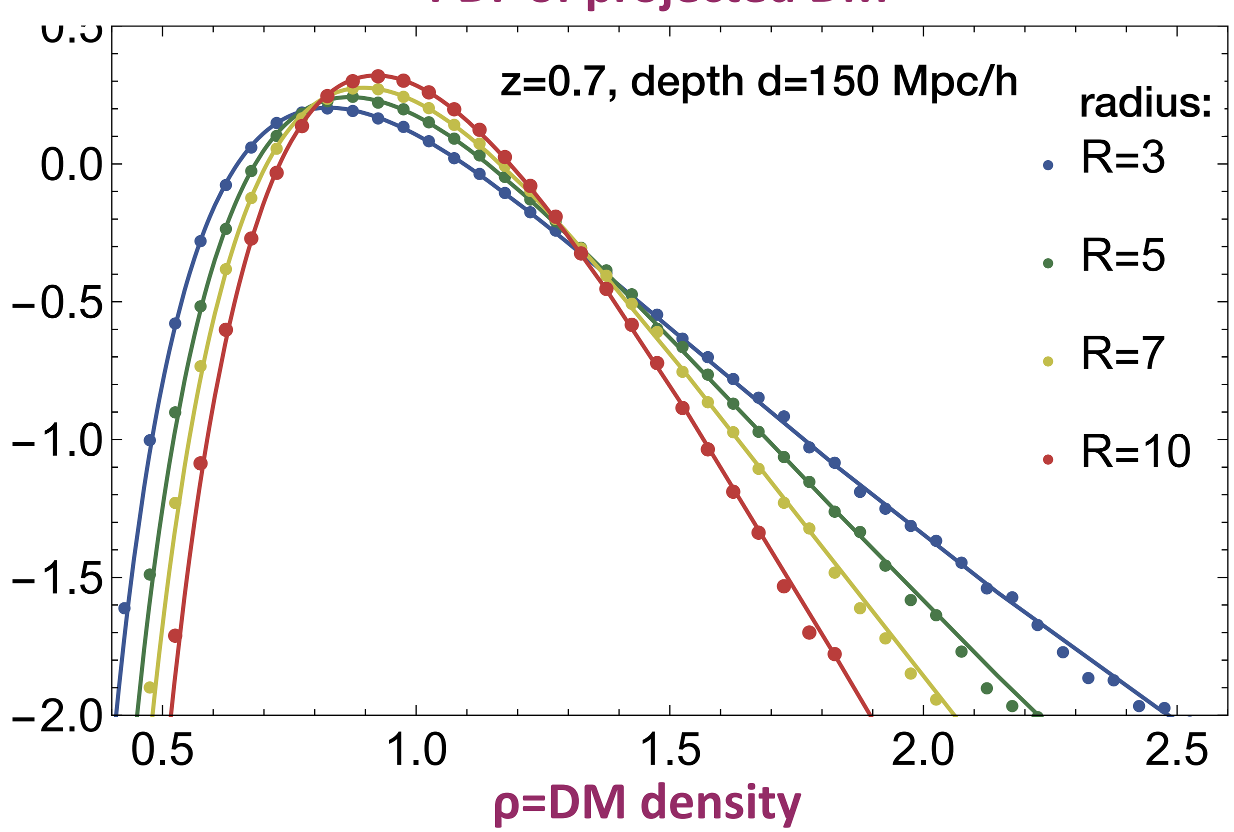
densities in redshift bins



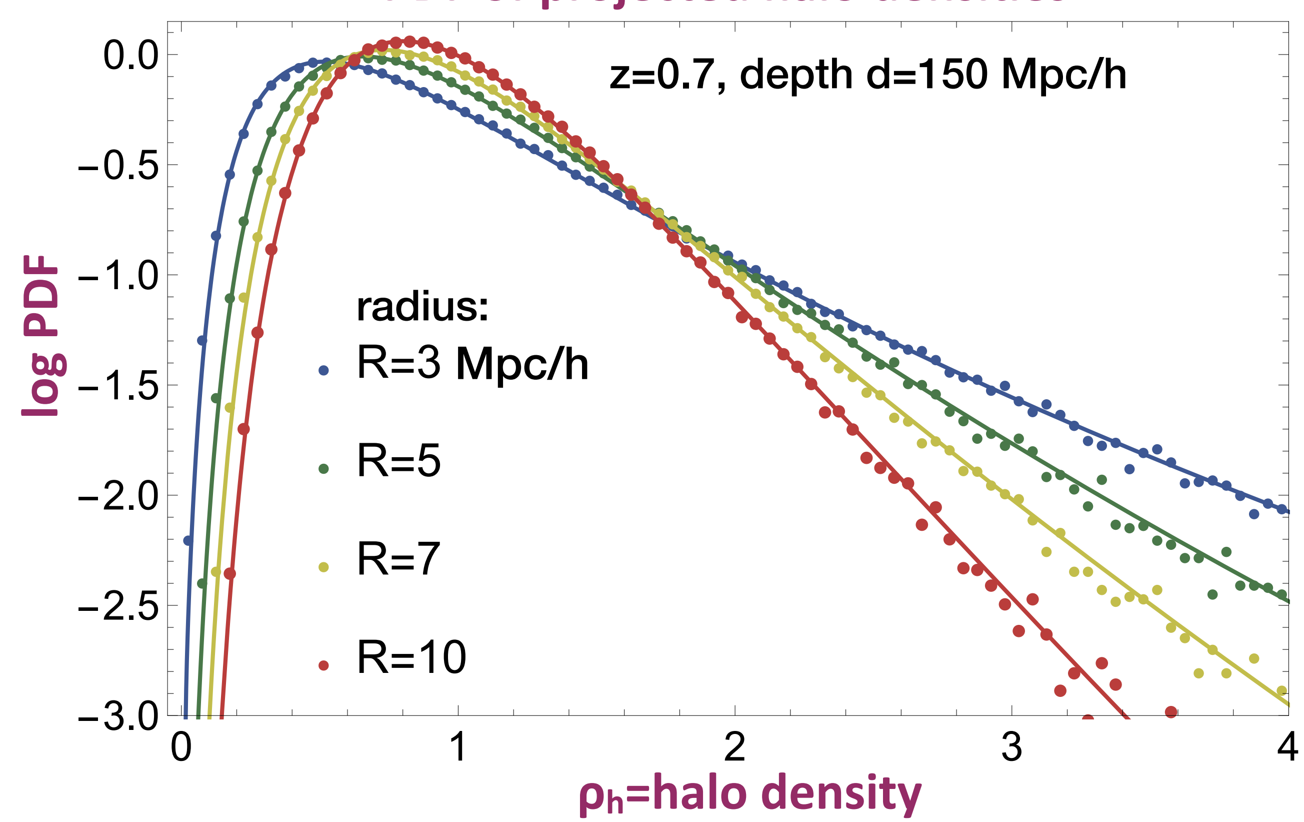
Densities in long cylinders: same formalism applies with cylindrical collapse

$$\zeta_{CC}(\tau_{2D}) = \left(1 - \frac{\tau_{2D}}{\nu}\right)^{-\nu}$$
$$\nu \approx 1.3$$

PDF of projected DM



PDF of projected halo densities



Conclusion

- Multi-scale density PDF can be predicted in the mildly non-linear regime with surprising accuracy (<1% for $\sigma = O(1)$) even in the rare event tails

- Predictions are fully analytical, parameter-free and explicitly cosmology-dependent

- Cosmic variance can be predicted from first principle

- We have an accurate model for biased density tracers, velocities, projected densities and (in progress) cosmic shear maps, including primordial non-Gaussianities

horizon-AGN

Large deviation principle:

an unlikely fluctuation is brought about by the least unlikely of all unlikely paths.