Advances in the perturbative description of LSS

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with:

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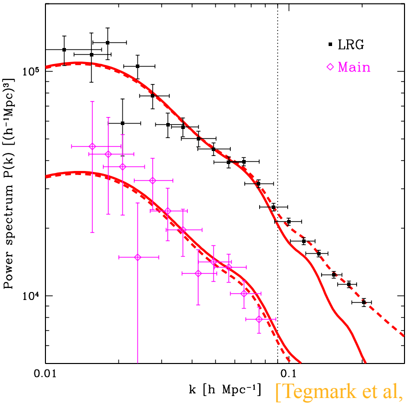
Understanding galaxy overdensity and shape clustering

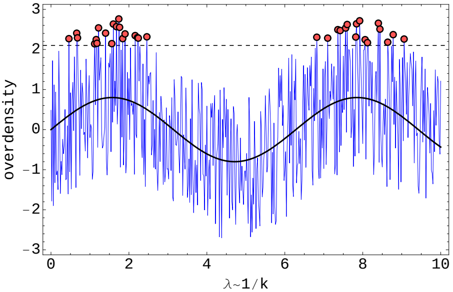
Galaxies and biasing of dark matter halos

Galaxies form at high density peaks of initial matter density:

- rare peaks exhibit higher clustering!



[Tegmark et al, 2006]



- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \ldots$
- Understanding bias is crucial for understanding the galaxy clustering

Earlier approaches to halo biasing

Local biasing model: halo field is a function of just DM density field

$$\delta_{\rm h} = c_\delta \delta + c_{\delta^2}\left(\delta^2 - \left\langle \delta^2 \right\rangle\right) + c_{\delta^3}\delta^3 + \ldots$$ [Fry & Gaztanaga, 1993]

Quasi-local (in space) relation of the halo density field to the dark matter [McDonald & Roy 2008, Assassi et al, 2014]

$$\begin{aligned}\delta_{\rm h}(\mathbf{x}) &= c_\delta \delta(\mathbf{x}) + c_{\delta^2}\delta^2(\mathbf{x}) + c_{\delta^3}\delta^3(\mathbf{x}) \\ &\quad + c_{s^2}s^2(\mathbf{x}) + c_{\delta s^2}\delta(\mathbf{x})s^2(\mathbf{x}) + c_\psi \psi(\mathbf{x}) + c_{st}s(\mathbf{x})t(\mathbf{x}) + c_{s^3}s^3(\mathbf{x}) \\ &\quad + c_\epsilon \epsilon + \ldots,\end{aligned}$$

with effective ('Wilson') coefficients c_l and variables:

$$s_{ij}(\mathbf{x}) = \partial_i\partial_j\phi(\mathbf{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta(\mathbf{x}), \qquad t_{ij}(\mathbf{x}) = \partial_i v_j - \frac{1}{3}\delta^{\rm K}_{ij}\theta(\mathbf{x}) - s_{ij}(\mathbf{x}),$$

$$\psi(\mathbf{x}) = [\theta(\mathbf{x}) - \delta(\mathbf{x})] - \frac{2}{7}s(\mathbf{x})^2 + \frac{4}{21}\delta(\mathbf{x})^2,$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

More complex structure (more physical effects) : [Senatore 2014, Mirbabayi et al 2014, Angulo et al 2015, Desjacques et al 2016]

Effective field theory of biasing

Non-local (time) and quasi-local (spece) relation of the halo density field to the dark matter

[Senatore 14, Angulo et al 15, Desjacques et al, 16]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\ H(t')\ \Big[\bar{c}_\delta(t,t')\ :\delta(\boldsymbol{x}_{\rm fl},t'):$$
$$+\ \bar{c}_{\delta^2}(t,t')\ :\delta(\boldsymbol{x}_{\rm fl},t')^2: +\bar{c}_{s^2}(t,t')\ :s^2(\boldsymbol{x}_{\rm fl},t'):$$
$$+\ \bar{c}_{\delta^3}(t,t')\ :\delta(\boldsymbol{x}_{\rm fl},t')^3: +\bar{c}_{\delta s^2}(t,t')\ :\delta(\boldsymbol{x}_{\rm fl},t')s^2(\boldsymbol{x}_{\rm fl},t'):+\dots$$
$$+\ \bar{c}_\epsilon(t,t')\ \epsilon(\boldsymbol{x}_{\rm fl},t') + \bar{c}_{\epsilon\delta}(t,t')\ :\epsilon(\boldsymbol{x}_{\rm fl},t')\delta(\boldsymbol{x}_{\rm fl},t'):+\dots$$
$$+\bar{c}_{\partial^2\delta}(t,t')\ \frac{\partial^2_{x_{\rm fl}}}{k_M^2}\delta(\boldsymbol{x}_{\rm fl},t')+\dots\Big]$$

Novice consideration of non-local in time formation, which depends on fields evaluated on past history on past path:

$$\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

Alternative - all effects chaptered in Lagrangian approach.

Note: Assembly bias effects captured in the scheme.

Effective field theory of biasing

Alternatively we can be similarly expand density of tracers as [Desjacques et al, 16]

$$\delta_{\mathrm{t}}(\boldsymbol{x}) = \sum_{O} c_o O_t(\boldsymbol{x}),$$

where we list operators O_h:

$$(1) \qquad \mathrm{tr}\left[\Pi^{[1]}\right],$$

$$(2) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2,$$

$$(3) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^3\right], \quad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right]\mathrm{tr}\left[\Pi^{[1]}\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^3, \quad \mathrm{tr}\left[\Pi^{[1]}\Pi^{[2]}\right].$$

where $\Pi^{[1]}_{ij}(\boldsymbol{k}) = \frac{k_i k_j}{k^2}\delta_m(\boldsymbol{k})$, with derivative operators

$$R_*^2 \nabla^2 \mathrm{tr}\left[\Pi^{[1]}\right], \ldots .$$

– series allows one to estimate the higher order (theory) errors

– coefficients - physics from the R_* scale - degeneracies

Effective field theory of biasing

Expansion of the field of galaxy shapes:

$$g_{ij}(\boldsymbol{x}) = \sum_{O} b_o O_{ij}(\boldsymbol{x}).$$

where the list of operators (up to higher derivatives and stochastic contributions) is

$$
\begin{aligned}
&(1) \qquad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}, \\
&(2) \qquad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}, \;\; \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&(3) \qquad \mathrm{TF}\left[\Pi^{[3]}\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[1]}\Pi^{[2]}\right]_{ij}, \;\; \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&\qquad\quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^3\right]_{ij}, \;\; \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \;\; \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2 \ldots
\end{aligned}
$$

Derivative operators relevant for leading power spectrum corrections

$$R_*^2 \nabla^2 \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}.$$

Projections onto the sky

Master observable correlators

$$P^{ab}_{ijlm}(k) = \langle \Pi^{\mathrm{t,a}}_{ij}(\boldsymbol{k})\Pi^{\mathrm{t,b}}_{lm}(\boldsymbol{k}')\rangle',$$

$$B^{abc}_{ijlmrs}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \langle \Pi^{\mathrm{t,a}}_{ij}(\boldsymbol{k}_1)\Pi^{\mathrm{t,b}}_{lm}(\boldsymbol{k}_2)\Pi^{\mathrm{t,c}}_{rs}(\boldsymbol{k}_3)\rangle'$$

Isotropy and homogeneity makes the expansion in spherical tensors useful

$$\Pi_{ij}(\boldsymbol{k}) = \frac{1}{3}\Pi^{(0)}_0(\boldsymbol{k})\delta^K_{ij} + \sum_{m=-2}^{2}\Pi^{(m)}_2(\boldsymbol{k})\boldsymbol{Y}^{(m)}_{ij}$$

different spectra are obtained by applying/subtracting the trace

$$\langle \delta^{\mathrm{t,a}}(\boldsymbol{k})\delta^{\mathrm{t,b}}(\boldsymbol{k}')\rangle' = P^{ab(0)}_{00}(k)$$

$$\langle \delta^{\mathrm{t,a}}(\boldsymbol{k})g^b_{ij}(\boldsymbol{k}')\rangle' = \boldsymbol{Y}^{(0)}_{ij}P^{ab(0)}_{02}(k)$$

$$\langle g^a_{ij}(\boldsymbol{k})g^b_{lm}(\boldsymbol{k}')\rangle' = \boldsymbol{Y}^{(0)}_{ij}\boldsymbol{Y}^{(0)}_{lm}P^{ab(0)}_{22}(k) + 2\sum_{q=1}^{2}\boldsymbol{Y}^{(q)}_{ij}\boldsymbol{Y}^{(-q)}_{lm}P^{ab(q)}_{22}(k)$$

bispectrum

$$\langle \delta^{\mathrm{t},a}(\boldsymbol{k}_1)g^b_{ij}(\boldsymbol{k}_2)g^c_{lm}(\boldsymbol{k}_3)\rangle = \boldsymbol{Y}^{(0)}_{ij}\left(\hat{k}_2\right)\boldsymbol{Y}^{(0)}_{lm}\left(\hat{k}_3\right)B^{abc,(0)}_{022}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \ldots,$$

Projections onto the sky

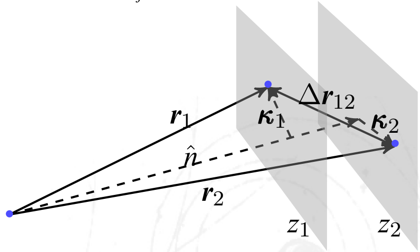
3D shape of galaxies get projected onto the onto the sky:

$$\gamma_{I,ij}(\boldsymbol{r},z) = \left(\mathcal{P}_{ik}(\hat{n})\mathcal{P}_{jl}(\hat{n}) - \frac{1}{2}\mathcal{P}_{ij}(\hat{n})\mathcal{P}_{kl}(\hat{n})\right) g_{kl}(\boldsymbol{r},z),$$

Using the helicity basis $(\hat{n}, \boldsymbol{m}_+, \boldsymbol{m}_-)$ intrinsic shape is:

$$\gamma_{I,ij}(\boldsymbol{r},z) = \gamma_{+2}(\boldsymbol{r},z)\boldsymbol{M}_{ij}^{(+2)} + \gamma_{-2}(\boldsymbol{r},z)\boldsymbol{M}_{ij}^{(-2)}.$$

$$\begin{aligned}
\{\xi, P\}^{ab}_{t,t} &= {}_{tt}\{\xi, P\}^{ab}, \\
\{\xi, P\}^{ab}_{t,\pm 2} &= \boldsymbol{M}_{ij}^{(\pm 2)}(\hat{n})\boldsymbol{\mathcal{P}}_{ijkl}(\hat{n})_{tg}\{\xi, P\}^{ab}_{ij}, \\
\{\xi, P\}^{ab}_{\pm 2,\pm 2} &= \boldsymbol{M}_{ij}^{(\pm 2)}(\hat{n})\boldsymbol{M}_{kl}^{(\pm 2)}(\hat{n}) \\
&\quad \times \boldsymbol{\mathcal{P}}_{ijmn}(\hat{n})\boldsymbol{\mathcal{P}}_{klrs}(\hat{n})_{gg}\{\xi, P\}^{ab}_{mnrs},
\end{aligned}$$



$$\boldsymbol{M}_{ij}^{(\pm 2)}(\hat{n})\boldsymbol{\mathcal{P}}_{ijkl}(\hat{n})\boldsymbol{Y}_{kl}^{(q)}(\hat{k}) = \begin{cases} |q| = 0, & \frac{1}{2}(1-\mu^2)e^{\pm 2i\phi}, \\ |q| = 1, & \pm\frac{i}{2\sqrt{2}}(\mathrm{sgn}(q) \mp \mu)\sqrt{1-\mu^2}e^{\pm 2i\phi}, \\ |q| = 2, & \frac{1}{2}(\mathrm{sgn}(q) \mp \mu^2)e^{\pm 2i\phi} \end{cases}$$

Effective field theory of biasing

Perturbative form of the shear tensor field is given in the form

$$\Pi^{\rm t}_{ij}(\boldsymbol{k}) = \sum_{n=1}^{\infty} (2\pi)^3 \delta^D_{\boldsymbol{k}-\boldsymbol{q}_{1n}} \mathcal{K}^{(n)}_{ij,\rm bias}(\boldsymbol{q}_1 \ldots, \boldsymbol{q}_n)\delta_L(\boldsymbol{q}_1)\ldots\delta_L(\boldsymbol{q}_n),$$

where kernels $\mathcal{K}^{(n)}_{ij,\rm bias}$ up to the third order are needed for one loop spectrum. We now apply the decomposition to the PT results up to one-loop power spectum

$$P^{ab,\rm one-loop}_{ijlm}(\boldsymbol{k}) = P^{ab,\rm lin}_{ijlm}(\boldsymbol{k}) + P^{ab,(22)}_{ijlm}(\boldsymbol{k}) + P^{ab,(13)}_{ijlm}(\boldsymbol{k}) + P^{ab,(31)}_{ijlm}(\boldsymbol{k}),$$

Linear, and loop (22), (13) contributions

$$P^{ab,\rm lin}_{ijlm}(\boldsymbol{k}) = \frac{k_i k_j k_l k_m}{k^4} c^{(a)}_{\Pi^{[1]}} c^{(b)}_{\Pi^{[1]}} P_{\rm lin}(k),$$

$$P^{ab,(22)}_{ijlm}(\boldsymbol{k}) = 2\,\mathcal{K}^{(2)}_{ij,a}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\mathcal{K}^{(2)}_{lm,b}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})P_{\rm lin}(q)P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|),$$

$$P^{ab,(13)}_{ijlm}(\boldsymbol{k}) = 3c^{(a)}_{\Pi^{[1]}} \frac{k_i k_j}{k^2} P_{\rm lin}(k)\,\mathcal{K}^{(3)}_{lm,b}(\boldsymbol{k},\boldsymbol{q},-\boldsymbol{q})P_{\rm lin}(q).$$

Effective field theory of biasing

New physical scale $k_M \sim 2\pi \left(\frac{4\pi}{3}\frac{\rho_0}{M}\right)^{1/3}$, which can be different then k_{NL}.

Interesting case $k_{NL} \gg k_M$!

We look at the correlations at $k \ll k_M$.

Each order in perturbation theory we get new bias coefficients:

$$O_{\rm t}(k,t) = \int_t \tilde{c}_{\delta,1}\left[D_t\delta^{(1)}(k) + \text{flow terms}\right] + \int_t \tilde{c}_{\delta,2}\left[D_t^2\delta^{(2)}(k) + \text{flow terms}\right] + \dots$$
$$= c_{\delta,1}\left[\delta^{(1)}(k) + \text{flow terms}\right] + c_{\delta,2}\left[\delta^{(2)}(k) + \text{flow terms}\right] + \dots$$

Emergence of degeneracy: choice of most convenient basis

Renormalization! (takes care of short distance effects at long distances)

In practice, $\tilde{c}_{\delta,1}$ is a bare parameter, the sum of a finite part and a counterterm:

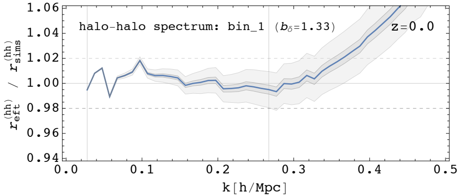
$$\tilde{c}_{\delta,1} = \tilde{c}_{\delta,1,\,\text{finite}} + \tilde{c}_{\delta,1,\,\text{counter}},$$

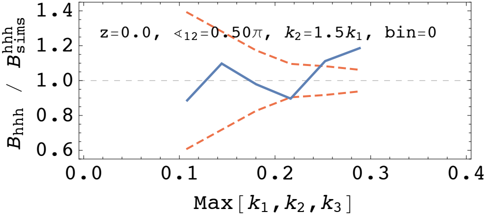
After renormalization we end up with 7(12) finite bias parameters b_i.

Observables: $P_{\rm tg}$, $P_{\rm gg}$, $B_{\rm ttt}$, $B_{\rm ttg}$, $B_{\rm tgg}$, $B_{\rm ggg}$

Effective field theory of biasing

Consistency with N-body simulations achieved up to the $k < 0.3\mathrm{Mpc}/h$ for the Power Spectra, similar for the Bispectrum $k < 0.15\mathrm{Mpc}/h$ [Angulo et al 2015]





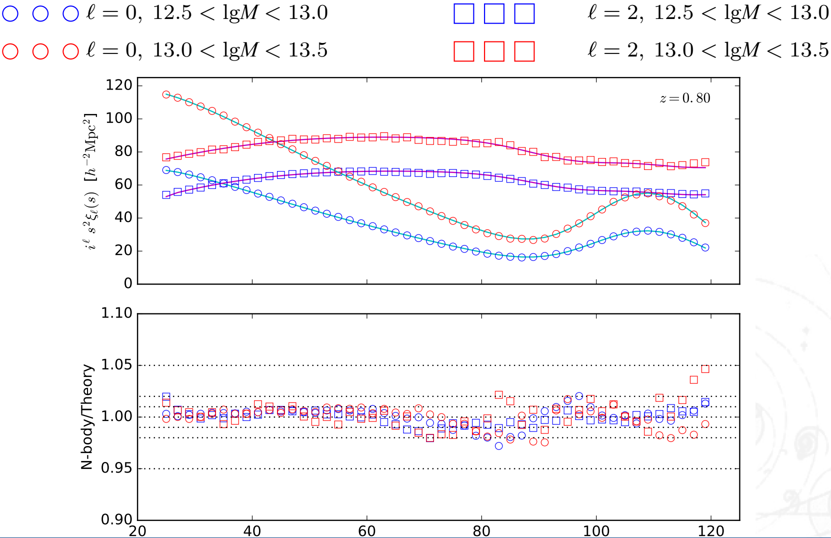
nLIT: k_{min}=0.04, k_{max}=0.15						
hm	hh	hmm	hhm	hhh	chi2	p
+	+	−	−	−	0.0804	1.000
+	+	+	−	−	0.719	0.9963
+	+	−	+	−	0.645	0.9998
+	+	−	−	+	0.747	0.9915
+	+	+	+	−	0.835	0.9746
+	+	+	−	+	1.08	0.1685
+	+	−	+	+	0.990	0.5345
+	+	+	+	+	1.08	0.1335

Most of the constraint comes form the 3-pt function

If we had the simulations for the 4-pt function 2-pt function would be fully predicted.

Bias in Lagrangian space in redshift space

DM halo multipoles multipoles in configurations space [with White and Castorina, 16]



Adding baryonic effects

- baryons at large distances described as additional fluid component (short distance physics is encoded in an effective stress tensor) [Angulo et al, 15]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\; H(t')\; \Big[\bar{c}_{\partial^2\phi}(t,t')\,\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\delta_b}(t,t')\,w_b\,\delta_b(\boldsymbol{x}_{{\rm fl}_b})$$

$$+\,\bar{c}_{\partial_i v^i_c}(t,t')\;w_c\,\frac{\partial_i v^i_c(\boldsymbol{x}_{{\rm fl}_c},t')}{H(t')} + \bar{c}_{\partial_i v^i_b}(t,t')\;w_b\,\frac{\partial_i v^i_b(\boldsymbol{x}_{{\rm fl}_b},t')}{H(t')}$$

$$+\,\bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\;\frac{\partial_i\partial_j\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2}\,\frac{\partial^i\partial^j\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \dots$$

$$+\,\bar{c}_{\epsilon_c}(t,t')\;w_c\,\epsilon_c(\boldsymbol{x}_{{\rm fl}_c},t') + \bar{c}_{\epsilon_b}(t,t')\;w_b\,\epsilon_b(\boldsymbol{x}_{{\rm fl}_b},t')$$

$$+\bar{c}_{\epsilon_c\partial^2\phi}(t,t')\;w_c\,\epsilon_c(\boldsymbol{x}_{{\rm fl}_c},t')\,\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\epsilon_b\partial^2\phi}(t,t')\;w_b\,\epsilon_b(\boldsymbol{x}_{{\rm fl}_b},t')\,\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2}\;\dots\Big]$$

where $\boldsymbol{x}_{\rm fl}$ is defined by Poisson equation and: [also, Yoo et al, 13, Blazek et al, 16, Schmidt, 16, Beutler et al, 17]

$$\boldsymbol{x}_{{\rm fl}_b}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\;\boldsymbol{v}_b(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))\,,\quad \boldsymbol{x}_{{\rm fl}_c}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\;\boldsymbol{v}_c(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

- similar expressions valid when including neutrinos, clustering dark energy …

Adding Non-Gaussianities

We assume that non-G. correlations are present only in the initial conditions and effect can be described by the squeezed limit, $k_L \ll k_S$ of correlation functions.

After horizon re-rentry, but still early enough to neglect all gravitational non-linearities, the primordial density fluctuation are given by

$$\delta^{(1)}(\boldsymbol{k}_S, t_{\rm in}) \simeq \delta_g(\boldsymbol{k}_S) + f_{\rm NL}\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in})\delta_g(\boldsymbol{k}_S - \boldsymbol{k}_L, t_{\rm in}) \;,$$

where $\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in}) = \frac{3}{2}\frac{H_0^2\,\Omega_m}{D(t_{\rm in})}\frac{1}{k_S^2\,T(k)}\left(\frac{k_L}{k_S}\right)^{\alpha}\delta_g(\boldsymbol{k}_L, t_{\rm in})$ and where $T(k)$ is the transfer function.

In the presence of primordial non-Gaussianities, additional components:

$$\begin{aligned}\delta_h(\boldsymbol{x},t) \simeq f_{\rm nl}\;\tilde{\phi}(\boldsymbol{x}_{\rm fl}(t,t_{\rm in}),t_{\rm in})\int^t dt'\;H(t')\;\left[\bar{c}^{\;\tilde{\phi}}(t,t') + \bar{c}^{\;\tilde{\phi}}_{\partial^2\phi}(t,t')\;\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \ldots\right] \\ + f_{\rm nl}^2\;\tilde{\phi}(\boldsymbol{x}_{\rm fl}(t,t_{\rm in}),t_{\rm in})^2\int^t dt'\;H(t')\;\left[\bar{c}^{\;\tilde{\phi}^2}(t,t') + \bar{c}^{\;\tilde{\phi}^2}_{\partial^2\phi}(t,t')\;\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \ldots\right] + \ldots\end{aligned}$$

Also studied in: [Assassi et al, 2015, Pier et al, 2016]

Nonlinear dynamics – including shell crossing

Lagrangian vs Eulerian framework

Eulerian:

Lagrangian:

Coordinate of a (t)racer particle at a given moment in time $\mathbf{r}$

$$\mathbf{r}(\mathbf{q},\tau) = \mathbf{q} + \psi(\mathbf{q},\tau),$$

is given in terms of Lagrangian displacement.

Continuity equation:

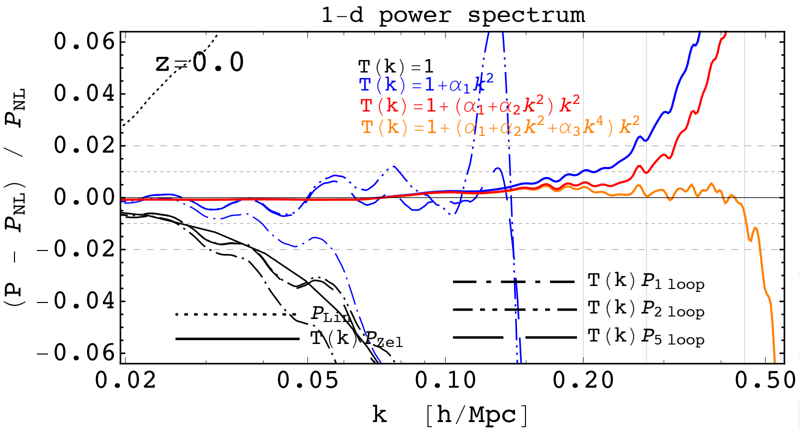
$$(1+\delta(\mathbf{r}))\, d^3r = d^3q \quad \text{vs.} \quad 1+\delta(\mathbf{r}) = \int_q \delta^D\left(\mathbf{r}-\mathbf{q}-\psi(\mathbf{q})\right),$$

Fourier space

$$(2\pi)^3\delta^D(\mathbf{k}) + \delta(\mathbf{k}) = \int_q e^{i\mathbf{k}\cdot\mathbf{q}}\ \exp\left(i\mathbf{k}\cdot\psi\right),$$

Clustering in 1D

1D case studied recently in: [McQuinn&White, '15, Vlah et al, '15]



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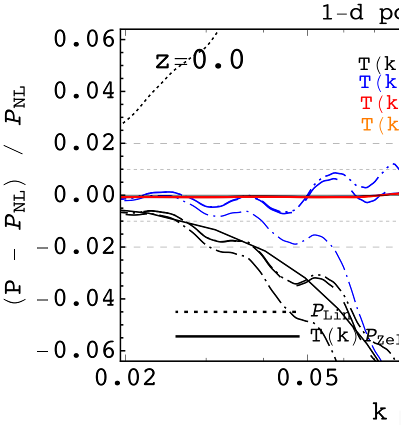
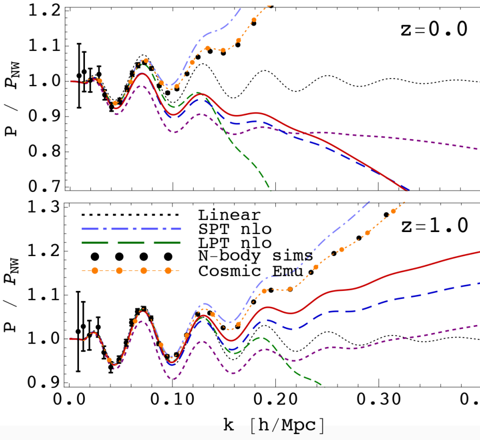




Path integrals and going beyond shell crossing

- as we saw the Lagrangian framework includes shell crossing
- Lagrangian dynamics can be compactly written using

$$\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi) = \boldsymbol{\epsilon},$$

where:

$$\phi \equiv (\psi, \upsilon)\,, \quad [\boldsymbol{L}_0]_{i_2 i_1} = \begin{pmatrix} \frac{\partial}{\partial\eta_2} & -1 \\ -\frac{3}{2} & \frac{\partial}{\partial\eta_2} + \frac{1}{2} \end{pmatrix}, \quad \boldsymbol{\Delta}_0(\phi) = \frac{3}{2}\left(0, \partial_{\boldsymbol{x}}\partial_{\boldsymbol{x}}^{-2}\delta + \psi\right).$$

Statistics of interest given by generating function

$$Z(\boldsymbol{j}) \equiv \int d\boldsymbol{\epsilon}\; e^{-\frac{1}{2}\boldsymbol{\epsilon}\boldsymbol{N}^{-1}\boldsymbol{\epsilon}+\boldsymbol{j}\phi[\boldsymbol{\epsilon}]} \;\text{ and }\; \langle\phi_{i_1}\phi_{i_2}\rangle = \frac{\partial^2}{\partial j_{i_1}\partial j_{i_2}} Z(\boldsymbol{j})\Big|_{\boldsymbol{j}=0},$$

which after the variable change becomes

$$Z(\boldsymbol{j}) \equiv \int d\phi\; e^{-S(\phi)+\boldsymbol{j}\phi},$$

with $S(\phi) = 1/2\,[\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi)]\,\boldsymbol{N}^{-1}\,[\boldsymbol{L}_0\phi + \boldsymbol{\Delta}_0(\phi)]\,.$ [McDonald&Vlah, '17]

Path integrals and going beyond shell crossing

We can organize our perturbation theory as:

$$S = S_g + S_p, \text{ where then we do } \exp(-S) = \exp(-S_g)(1 - S_p + S_p^2/2 + ...)$$

where we can choose what the "Gaussian part" will be, i.e.

$$S_g \equiv 1/2\chi\boldsymbol{N}\chi + i\chi[W^{-1}\boldsymbol{L}_0]\phi \equiv 1/2\chi\boldsymbol{N}\chi + i\chi\boldsymbol{L}\phi$$

and

$$S_p \equiv i\chi\boldsymbol{\Delta}_0(\phi) + i\chi[(1 - W^{-1})\boldsymbol{L}_0]\phi \equiv i\chi\boldsymbol{\Delta}(\phi),$$

where χ is the auxiliary field from the Hubbard-Stratonovich transformation.

Perturbation theory result : $Z(\boldsymbol{j}) = Z_0(\boldsymbol{j}) + Z_1(\boldsymbol{j}) + \ldots$

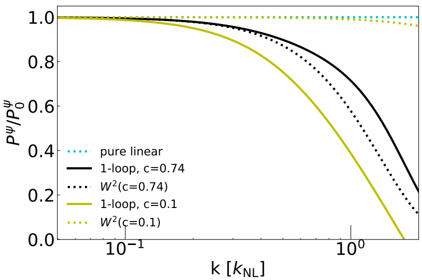
Leading order result: truncate Zel'dovich dynamics!!!

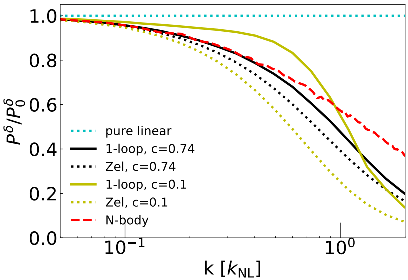
$$Z_0 = e^{\frac{1}{2}\boldsymbol{j}.\boldsymbol{C}.\boldsymbol{j}} \text{ and } P(k) = \int d^3q \; e^{i\boldsymbol{q}\cdot\boldsymbol{k}} e^{-\frac{1}{2}k_ik_jA^W_{ij}}$$

higher orders more complicated, build in renormalization! [McDonald&Vlah, '17]

Path integrals and going beyond shell crossing

$W = \exp(-ck^2),\ n = 0.5$





Significance and connection EFT formalism:

- no need of EFT free parameters, i.e. counter terms are predicted
- CMB lensing: direct information on baryonic and neutrinos physics
- reduction of degeneracy in galaxy bias coefficients
- possible connection to the EFT formalism by matching the $k \to 0$ limit

Summary

Key points:

- Shell crossing can be consistently added to the perturbative Lagrangian scheme.
- EFT framework is viable for study clustering of shapes as well as overdensities of galaxies.
- It offers most simplifications on largest scales & Lagrangian setting is a natural for the study of BAO effects in LSS statistics..

Wiggles for halos in redshift rpace

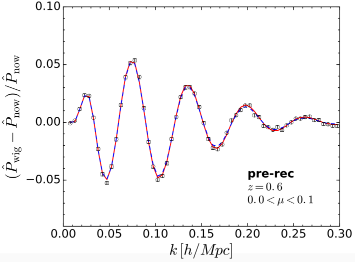
$$P(\boldsymbol{k}) = \int_q e^{-i\boldsymbol{q}\cdot\boldsymbol{k}}\Big(1-\text{bias}\Big)\exp\left(-\frac{1}{2}A^s(\boldsymbol{k},\boldsymbol{q})\right)\Bigg|_{\lambda_1=\lambda_2=0} + \text{h.o.} + \text{``stochastic''},$$

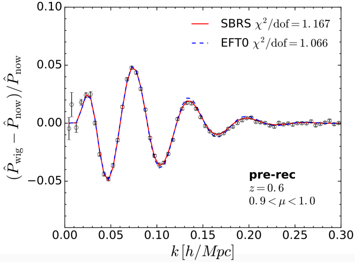
where we e.g. $A^s(\boldsymbol{k},\boldsymbol{q}) = \left\langle \Big(\lambda_1\delta_L(\boldsymbol{q}_1)+\lambda_2\delta_L(\boldsymbol{q}_2)+\boldsymbol{k}\cdot\Delta^s(\boldsymbol{q})\Big)^2\right\rangle_c$, gives [with Ding, Seo, et. al.]

$$\delta P(k,\nu) = e^{-k^2(1+f(2+f)\nu^2)\Sigma^2(q_{\max})}\left(b_1^2+2fb_1\nu^2+f^2\nu^4+b_\partial\left(b_1+f\nu^2\right)\frac{k^2}{k_L^2}\right)\delta P_L(k,\tau)+\text{h.o.}$$

where $q_{\max}$ implicitly given by $\frac{\partial}{\partial q}\left[\Big(1-i\hat{c}_q(\partial_{\lambda_1}+\partial_{\lambda_2})-\hat{c}_q^2\partial_{\lambda_1}\partial_{\lambda_2}\Big)\delta\mathcal{A}^s(\boldsymbol{k},\boldsymbol{q})\right]_{\substack{\lambda_1=\lambda_2=0\\ q=q_{\max}}} = 0.$

depends on k,ν as well as bias parameters $c_\delta, c_{\partial^2\delta},\ldots$ simplest $\Sigma^2 = \int\frac{dp}{3\pi^2}(1-j_0(qk))P_L(p)$.





Wiggles for halos in redshift rpace

Results and parameters estimate:

[with Ding, Seo, et. al.]

	Pre-reconstruction
EFT0 model	Free: $\alpha_\perp$, $\alpha_\parallel$, f, b_1, b_∂ Fixed: Σ_{xy}, $\Sigma_z (= (1+f_{\rm fid})\Sigma_{xy})$. For matter, $b_1 = 1$.
EFT1 model	Free: $\alpha_\perp$, $\alpha_\parallel$, Σ_{xy}, f, b_1, b_∂. Note* $\Sigma_z = (1+f_{\rm fid})\Sigma_{xy}$ Fixed: for matter, $b_1 = 1$.